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## Transaction Correctness

**Enterprise Scale Data Management**

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### Schedules and Histories

**Definition 3.1 (Schedules and histories):**  
Let  $T = \{t_1, \dots, t_n\}$  be a set of transactions, where each  $t_i \in T$  has the form  $t_i = (op_i, <_i)$  with  $op_i$  denoting the operations of  $t_i$  and  $<_i$  their ordering.

(i) A **history** for  $T$  is a pair  $s = (op(s), <_s)$  s.t.

- (a)  $op(s) \subseteq \bigcup_{i=1..n} op_i \cup \bigcup_{i=1..n} \{a_i, c_i\}$
- (b) for all  $i, 1 \leq i \leq n$ :  $c_i \in op(s) \Leftrightarrow a_i \notin op(s)$
- (c)  $\bigcup_{i=1..n} <_i \subseteq <_s$
- (d) for all  $i, 1 \leq i \leq n$ , and all  $p \in op_i$ :  $p <_s c_i$  or  $p <_s a_i$
- (e) for all  $p, q \in op(s)$  s.t. at least one of them is a write and both access the same data item:  $p <_s q$  or  $q <_s p$

(ii) A **schedule** is a prefix of a history.

**Definition 3.2 (Serial history):**  
A history  $s$  is **serial** if for any two transactions  $t_i$  and  $t_j$  in  $s$ , where  $i \neq j$ , all operations from  $t_i$  are ordered in  $s$  before all operations from  $t_j$  or vice versa.

Transactional Information Systems
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### History Example

r1[x]  
w2[y]  
w3[y]  
r1[z]

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w1[x]  
r3[z]  
w3[z]

r2[x]

r1[x]r2[x]r1[z]w1[x]w2[y]r3[z]w3[y]w3[z]c1c2c3

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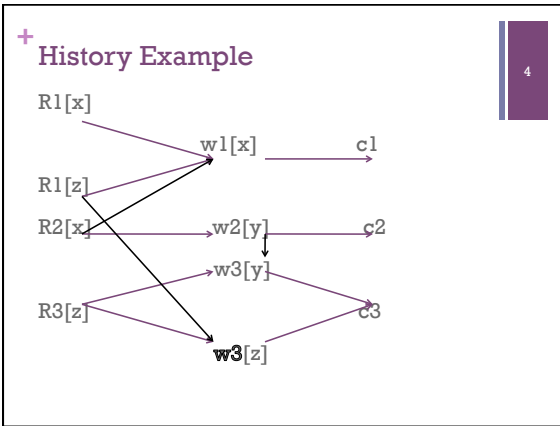
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- + Histories**
- Without loss of generality:
    - Examples will be total orders
  - Notations:
    - Trans(H): transactions in H
    - Commit(H): committed in H
    - Abort(H): aborted in H
    - Active(H): not committed and not aborted.
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- + Correctness**
- Function  $\sigma : S \rightarrow \{0,1\}$  such that  $\text{correct}(S) = \{s \text{ in } S \mid \sigma(s)=1\}$
  - Pragmatic considerations:
    - $\text{Correct}(S) \neq \emptyset$
    - $\text{Correct}(S)$  is efficiently decidable
    - $\text{Correct}(S)$  is sufficiently large (WHY?)
  - Goal: develop several such criteria given that semantics not known.
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**+ Correctness**

- Syntactical semantics for schedules based on an intuitive notion:
  - Each transaction is a correct mapping, i.e.,



Hence, serial execution of transactions will be correct.

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**+ General Idea**

- Notion of equivalence of two schedules S1 and S2
- Use this notion of equivalence to accept all schedules which are “equivalent” to some serial schedule as being correct.
- How to establish this equivalence notion?

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**+ Semantics**

- Equivalence via a notion of semantics:
  - We do not know the semantics of transaction programs
  - We need a general notion that can capture all potential transaction semantics
- Need a general enough and powerful notion that can capture all possible semantics of transactions.

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**+ Herbrand Semantics** 10

- Read operation  $ri[x]$  reads the last value by the last write that occurs before  $ri[x]$ .
- $Wi[x]$  writes a value that potentially depends on the value of all data items that  $T_i$  has read prior to  $wi[x]$ .

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**+ Herbrand Semantics** 11

- Abstract notion of semantics:
  1.  $ri[x]$  reads the last  $wj[x]$  ( $j \neq i$ ) before  $ri[x]$ .
  2.  $Wi[x]$  depends on:
    1. Data from DB
    2. Transactions in ACTIVE U COMMIT prior to  $wi[x]$ .

Last write is well defined!!! Why?

Assumption I: No transaction Aborts  $wi[x] \rightarrow Rk[x]$   
 Assumption II: Initial Transactions:  
 $w0[\text{entire-database}]$ , or equivalently  
 $w0[x, y, z, \dots]$

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**+ Formal Definition: H-Semantics** 12

- $Hs(ri[x]) = Hs(wj[x])$  where  $wj[x]$  is the last write operation
- $Hs(Wi[x]) = \text{fix}(Hs(ri[y_1]), \dots, Hs(ri[y_m]))$
- HU (Herbrand Universe) for transaction: what is conveyed to the transaction.
- HS for schedules: what is the permanent effect of the schedule of transactions.

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**+ Example** 13

- $S = w_0[x]w_0[y]c_0r_1[x]r_2[y]w_2[x]w_1[y]c_1c_2$
- $Hs[w_0[x]] = f_0x()$
- $Hs[w_0[y]] = f_0y()$
- $Hs[r_1[x]] = f_0x()$
- $Hs[r_2[y]] = f_0y()$
- $Hs[w_2[x]] = f_2x(Hs[r_2[y]]) = f_2x(f_0y())$
- $Hs[w_1[y]] = f_1y(Hs[r_1[x]]) = f_1y(f_0x())$

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**+ Herbrand Universe** 14

Let  $D = \{x, y, z, \dots\}$  be a finite set of data items. For a transaction  $T$  let  $op(T)$  denote all the steps of  $T$ . The HU of  $T_i$  is:

- $f_0x()$  in HU for each  $x$  in  $D$
- If  $w_i[x]$  in  $Op(T_i)$  then  $fix(v_1, \dots, v_m)$  in HU where  $v_i$  are the values read by  $T_i$  before  $w_i[x]$ .

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**+ History Semantics** 15

- $H[h]: D \rightarrow HU$
- $H[h](x) := Hs(w_i[x])$

Where  $w_i(x)$  is the last operation in  $h$  writing  $x$ .

In other words – the semantics of a history  $h$  is the set of values that are written last in  $h$ .

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**+ Why are we doing all this?**

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- General/abstract notion of semantics.
- Can work with any interpretation of the transaction program, i.e., **we do not have to worry about the program semantics of the transaction as to how they manipulate the data.**

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**+ Example**

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- $h = w_0[x]w_0[y]c_0r_1[x]r_2[y]w_2[x]w_1[y]c_2c_1$
- $Hs[x] = Hs[w_2[x]] = f_2x(f_0y())$
- $Hs[y] = Hs[w_1[y]] = f_1y(f_0x())$

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**+ Final State Equivalence**

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- S and S' over the same set of transactions then S is equivalent to S' if  $H(S) = H(S')$ .

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**+ Example** 19

- $S = r1[x]r2[y]w1[y]r3[z]w3[z]r2[x]w2[z]w1[x]$
- $S' = r3[z]w3[z]r2[y]r2[x]w2[z]r1[x]w1[y]w1[x]$
- $H[S](x) = f1x(f0x()) = H[S'](x)$
- $H[S](y) = f1y(f0x()) = H[S'](y)$
- $H[S](z) = f2z(f0x(), f0y()) = H[S'](z)$

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**+ Another Example** 20

- $S = r1[x]r2[y]w1[y]w2[y]c1c2$
- $S' = r1[x]w1[y]r2[y]w2[y]c1c2$
- $H[S](y) = f2y(f0y())$
- $H[S'](y) = f2y(H[S'](r2[y])) = f2y(f1y(f0x()))$

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**+ Observations** 21

- Example shows that we cannot simply determine equivalence on final write operation.
- What preceded must also be taken into account.
  - In S: final value of y is based on initial value of y.
  - In S': final value of y is based on the value of y written by T1.
- Our task: can we build an efficient tool to determine equivalence *efficiently*?

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**+ Reads-from Relation, Useful, Alive, and Dead Steps** 22

- $Rj[x]$  reads-x-from  $wi[x]$  if  $wi[x]$  is the last write such that  $wi[x] < rj[x]$ .
- $RF(S) = \{ (Ti, x, Tj) \mid rj[x] \text{ reads-x-from } wi[x] \}$
- Step  $p$  is directly useful for  $q$  denoted  $p \rightarrow q$  if:
  - $Q$  reads-from  $P$  or
  - $P$  is a read step and  $q$  is a subsequent write in the same transaction.
- $\rightarrow^*$  is the transitive closure of  $\rightarrow$

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**+ Reads-from Relation, Useful, Alive, and Dead Steps** 23

- $P$  is alive in  $S$  if it is useful for some step in  $T^\infty$ :
  - Exists  $q$  in  $T^\infty$  such that  $p \rightarrow^* q$
  - Otherwise  $P$  is dead in  $S$ .
- Live reads-from relation:
  - $LRF(S) = \{ (Ti, x, Tj) \mid rj[x] \text{ is alive and } rj[x] \text{ in } RF(S) \}$

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**+ Example** 24

- $S = w0[x,y]r1[x]r2[y]w1[y]w2[y]r^\infty[x,y]$
- $S' = w0[x,y]r1[x]w1[y]r2[y]w2[y]r^\infty[x,y]$
- $RF(S) = \{ (T0, x, T1), (T0, y, T2), (T0, x, T^\infty), (T2, y, T^\infty) \}$
- $RF(S') = \{ (T0, x, T1), (T0, y, T2), (T0, x, T^\infty), (T2, y, T^\infty) \}$
- $R2[y]$  alive in  $S$  and  $S'$  (verify)
- $R1[x]$  dead in  $S$  but alive in  $S'$  (verify)

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**+ Example (contd.)** 25

- $LRF(S) = \{(T0, y, T2), (T0, x, T^\infty), (T2, y, T^\infty)\}$
- $LRF(S') = RF(S')$
  
- Redefine FSE: S and S' are final state equivalent if and only if  $LRF(S) = LRF(S')$  (Prove it – omitted).
  
- Build a tool that will allow to “efficiently” identify the LRF relations: STEP GRAPH.

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**+ Step Graph Construction** 26

- Construct step graph  $D(S) = (V, E)$  where:
  - $V = op(S)$
  - $E = \{(p, q) \mid p, q \text{ in } V \text{ and } p \rightarrow q\}$
  
- It can be shown that  $LRF(s) = LRF(s')$  iff  $D(s) = D(s')$ .
  
- S f.s.e. S' iff  $D(S) = D(S')$  and  $op(S) = op(S')$

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**+ Examples to check FSE using Step Graph** 27

- $S = r1[x]r2[y]w1[y]r3[z]w3[z]r2[x]w2[z]w1[x]$
- $S' = r3[z]w3[z]r2[y]r2[x]w3[z]r1[x]w1[y]w1[x]$
  
- Construct  $D(S)$  and  $D(S')$  in class.

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**+ Another Example**

- $S = r1[x]r2[y][w1[y]w2[y]$
- $S' = r1[x]w1[y]r2[y]w2[y]$

- Construct  $D(S)$  and  $D(S')$  to check FSE.

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**FSR: Example 3.9**

$s = r_1(x) r_2(y) w_1(y) w_2(y)$        $s' = r_1(x) w_1(y) r_2(y) w_2(y)$

D(s):

D(s')

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**+ Testing for FSE**

- FSE can be decided in time polynomial in the length of two schedules.
- FSR: A history is FSR if there exists a serial history  $S'$  such that  $S$  is FSE to  $S'$ .
- $S = r1[x]r2[y]w1[y]r3[z]w3[z]r2[x]w2[z]w1[x]$   
Is equivalent to serial history T3-T2-T1 (verify)

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**+ Testing for FSR** 31

- How to test for FSR:
  - Try all  $N!$  serializations of  $N$  transactions.
- Not Efficient!!!
- More importantly: lets revisit our examples of Lost Update and Fund Transfer and see if it works from application point-of-view?

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**+ Lost Update** 32

- History corresponding to lost update:
  - $H=r1[x]r2[x]w1[x]w2[x]$
  - Possible serializations:  $H1=r1[x]w1[x]r2[x]w2[x]$   
OR  $H2=r2[x]w2[x]r1[x]w1[x]$
- Construct  $D(H)$ ,  $D(H1)$  and  $D(H2)$  and see if this  $H$  is not FSE either to  $H1$  or  $H2$ ?

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**+ Fund Transfer** 33

- Fund Transfer History:
  - $H=r2[x]w2[x]r1[x]r1[y]r2[y]w2[y]$
  - FSE to both  $T1-T2$  and  $T2-T1$ .
- Even if we can develop an efficient tool to enforce FSR executions, it is not good enough for our purpose.

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**+ Key Insight**

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- We need to strengthen the notion of final state serializability:
  - By not only focusing on the state of the database
  - But also requiring that the “database view” observed by each transaction in the equivalent schedules is identical.

NEXT LECTURE.

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