

Homework 1, 290D, Fall 07

Partial solution set

Problem 2 (10 points): Consider the edit distance between two strings defined as the minimum number of inserts, deletes, and replacement operations needed to transform one string into another. Show that this distance is a metric.

Triangle inequality sketch: Consider the space of all strings. Define undirected edges on this graph between nodes which are at a distance of 1 away in terms of reachability by a single operation of insertion/deletion/replacement. The distance between two strings is defined as the shortest path between them. Triangle inequality follows from the property of shortest paths.

Problem 6 (20 points): Suppose vector space V has a dimension of k . Prove that

- any k independent vectors in V form a basis
- any k vectors that span V form a basis.

6 (a) Let $X = \{x_1, x_2, x_k\}$ be a given set of independent vectors. We need to show that X spans V .

Proof sketch 1: Since V has a dimension of k , there exists a basis set $Y = \{y_1, y_2, \dots, y_k\}$. Assume that z is a vector that is not spanned by X , i.e., z cannot be expressed as a linear combination of X . Write each x_i as a linear combination of Y . Thus, z cannot be expressed as a linear combination of linear combination of Y . This contradicts the assumption that Y forms a basis.

Proof sketch 2: Assume that z is a vector that is not spanned by X . Then, $X \cup \{z\}$ forms a linearly independent set. This implies that the dimension of V is $k+1$, a contradiction.

6 (b) Let $X = \{x_1, x_2, x_k\}$ be a given set of vectors that span V . We need to show that X is linearly independent.

Assume X is not linearly independent. Reduce X to a linearly independent set X' of $m < k$ vectors. X' forms a basis. This implies that the dimension of V is less than k , a contradiction