In the problems below, the labeling functions for transition systems are defined as: $L : S \rightarrow 2^{AP}$ instead of $L' : S \times AP \rightarrow \{\text{true}, \text{false}\}$ where $p \in L(s) \Leftrightarrow L'(s, p) = \text{true}$.

1. Consider the following transition system:

   $M = (AP, S, R, I, L)$ with the set of states $S = \{0, 1, 2, 3, 4\}$, the initial set of states $I = \{0, 1\}$, the transition relation $R = \{(0, 2), (0, 3), (1, 3), (2, 3), (3, 1), (3, 2), (3, 4), (4, 3), (4, 4)\}$, the set of atomic propositions $AP = \{p, q, r\}$ and the labeling function $L : S \rightarrow 2^{AP}$ where $L(0) = \{p\}$, $L(1) = \{r\}$, $L(2) = \{q, r\}$, $L(3) = \{q\}$, and $L(4) = \{p, q, r\}$.

   Which of the following LTL formulas hold for the above transition system? For the properties that fail, give a counter-example execution path.

   $Xq, G(p \lor q), Fr, pUr, FGq, GFr, (X(\neg r)) \Rightarrow (XXr), G(q \lor Xq), pU(G(q \lor r)), (XXq)U(q \lor r)$

2. Assume that $p$, $q$ and $r$ are atomic properties.

   (a) Write LTL formulas that correspond to the following properties: 1) $p$ holds only a finite number of times, 2) if $p$ holds sometime, then $q$ must hold at least once before it, 3) $p$ and $q$ become true in strictly alternating fashion, 4) Whenever $p$ holds $q$ must hold in the next state, 5) $q$ holds infinitely many times.

   (b) Write CTL formulas that correspond to the following properties: 1) From each reachable state it is possible to reach a state where $p$ holds, 2) Whenever $p$ holds, eventually $q$ will hold, 3) Eventually, a path on which $p$ always holds will be reached, 4) It is possible for $p$ and $q$ to hold at the same time, 5) Whenever $p$ holds, eventually $r$ will hold and $q$ will hold until $r$ holds.

3. (a) Show that all CTL formulas can be written using only the temporal operators EX, EU and AU. I.e., for each of the CTL formulas $AXp$, $AGp$, $EGp$, $AFp$, $EFp$, write an equivalent formula that only contains the temporal operators EX, EU and AU and Boolean operators and atomic properties.

   (b) Show that temporal operators $AGp$, $EGp$, $AFp$, $EFp$, $pEUq$ and $pAUq$ can be written by using only themselves, temporal operators AX and EX, Boolean operators, and atomic properties. For example, you should write a temporal logic formula that is equivalent to $AGp$ that uses AG, and possibly temporal operators AX or EX, Boolean operators $\neg, \land, \lor$ and atomic property $p$. As you might suspect stating the trivial identity (i.e., $AGp \equiv AGp$) is not an acceptable answer.

   (c) Assume that the next-until operator $XU$ is defined as follows:

   $$pXUq = X(pUq)$$
Show that all LTL formulas can be written using only the temporal operator XU and boolean operators and constants. I.e., for each of the LTL formulas Xp, pUq and Gp write an equivalent formula that only contains the temporal operator XU and Boolean operators and atomic properties.

4. (a) Give a transition system that satisfies only one of the following two properties: FGp, AFAGp.

(b) Give a transition system that satisfies only one of the following two properties: FGp, AFEGp.

(c) Give a transition system that satisfies only one of the following two properties: FGp, EFEGp.

(d) Give a transition system that satisfies only one of the following two properties: GFp, AGEFp.

5. Consider the following transition system: \( M = (AP, S, R, I, L) \) with the set of states \( S = \{0, 1, 2, 3, 4\} \), the initial set of states \( I = \{0\} \), the transition relation \( R = \{(0, 1), (1, 2), (1, 4), (2, 3), (3, 3), (4, 4)\} \), the set of atomic propositions \( AP = \{p, q, r\} \) and the labeling function \( L : S \rightarrow 2^{AP} \) where \( L(0) = \{p\} \), \( L(1) = \{p, q\} \), \( L(2) = \{q\} \), \( L(3) = \{r\} \) and \( L(4) = \{q\} \).

Show the steps of the CTL model checking algorithm (by labeling the states with the subformulas that hold in that state) while evaluating the following CTL formulas on this transition system: EFEGr, AG(p ∨ q), AF(r ∨ q) and pEU(AGp). Which of these formulas hold for this transition system?

6.

```plaintext
boolean a, b;
initial: a = false and b = false;

P1 :: while True do
  1:   a := true;
  2:   b := not a;
endwhile

P2 :: while True do
  1:   b := true;
  2:   a := not b;
endwhile

P = cobegin P1 || P2 coend
```

(a) Draw the transition system for the above concurrent program. The variables a and b are shared, the assignments are atomic and each process loops between two program points (1 and 2).
(b) For each of the following temporal formulas show the states which satisfy them (for the formulas that contain more than one temporal operator also show the states that satisfy the subformula). Which of these temporal formulas are satisfied by the whole transition system:

\[
\begin{align*}
&\text{AG}(\neg a \lor \neg b) \\
&\text{EG}(\neg a \lor \neg b) \\
&\text{AG}(\text{AF}(a = b)) \\
&\text{AG}(\text{AF}(a \neq b)) \\
&\text{AG}((pc_1 = pc_2 = 2) \Rightarrow (a = b)) \\
&\text{AG}((pc_1 = pc_2 = 2) \Rightarrow \text{AF}(a)) \\
&\text{AG}((pc_1 = pc_2 = 2) \Rightarrow \text{AX}(a \neq b)) \\
&\text{AG}((pc_1 = pc_2 = 2) \Rightarrow \text{EX}(\neg a))
\end{align*}
\]