CS 267 – Spring 2008 – Homework Assignment 1

Due Wednesday, April 16, in class.

Do not discuss the problems with anyone other than the instructor.

1. Consider the following transition system $M = (AP, S, R, S_0, L)$ with the set of states $S = \{0, 1, 2, 3\}$, the initial set of states $S_0 = \{0\}$, the transition relation $R = \{(0, 1), (0, 3), (1, 1), (1, 2), (2, 3), (3, 2)\}$, the set of atomic propositions $AP = \{p, q\}$ and the labeling function $L : S \to AP$ where $L(0) = \{p\}$, $L(1) = \{q\}$, $L(2) = \{p, q\}$, and $L(3) = \{p\}$.

(a) For each of the following temporal formulas show the states which satisfy them:

$$\mathrm{EG}p,\ \mathrm{EG}q,\ \mathrm{AG}p,\ \mathrm{EF}p,\ \mathrm{AF}p,\ \mathrm{AF}q,\ p\mathrm{EU}q,\ p\mathrm{AU}q, \mathrm{AG}(p \Rightarrow \mathrm{AF}q), \mathrm{AG}(q \Rightarrow \mathrm{AF}p).$$

(b) Which of these properties are satisfied by the transition system M? For the ACTL properties (the properties that only contain the path quantifier A) that are not satisfied by M give a counter-example execution path.

(c) Based on the above transition system, show the iterative fixpoint computations (show the set of states for each iteration) and the results for the following μ-calculus formulas:

$$\nu z\ .\ (p\ \wedge\ \mathrm{EX}z))$$
$$\mu z\ .\ (q\ \vee\ (p\ \wedge\ \mathrm{AX}z))$$
$$\mu y\ .\ \nu z\ .\ ((q\ \wedge\ \mathrm{EX}z)\ \vee\ \mathrm{EX}y)$$

2. Assume that the next-until operator XU is defined as follows:

$$p\mathrm{XU}q = \mathrm{X}(p\mathrm{U}q)$$

Show that the LTL formulas containing temporal operators X, U, and G can be converted to formulas that contain only the temporal operator XU and boolean operators and constants. I.e., for each of the LTL formulas Xp, pUq and Gp write an equivalent formula that only contains the temporal operator XU and boolean operators and constants.

3. Prove or disprove the following properties. For a disproof show a transition system and a state which satisfies one of the properties but not the other. For a proof, you need to show the equivalence of the two formulas based on the temporal logic semantics.

(a) $\mathrm{AGF}p \stackrel{?}{=} \mathrm{AGEF}p$

(b) $\mathrm{AGF}p \stackrel{?}{=} \mathrm{AGAF}p$

(c) $\mathrm{AFG}p \stackrel{?}{=} \mathrm{AFAG}p$