CS 267 – Spring 2008 – Homework Assignment 4
Due Monday, June 9.
Do not discuss the problems with anyone other than the instructor.

1. Consider a transition system with two states encoded using a single boolean variable x, where the initial state is $\neg x$ and the transition relation is defined as $x \lor \neg x \land x'$.

(a) Show the Boolean logic formula that corresponds to unrolling the above transition system twice starting from the initial state.

(b) Assume that we would like to check the property EX(EX($\neg x$)). Using the result from part (a), construct a boolean logic formula F that is satisfiable if the initial state satisfies the property. Give a satisfying assignment to the variables in the formula F if F is satisfiable.

(c) Now, assume that we would like to check the property EF(x). Again, using the result from part (a), construct a boolean logic formula F that is satisfiable if the initial state satisfies the formula EF(x) within two steps of execution. Give a satisfying assignment to the variables in the formula F if F is satisfiable.

2. You are given the following program:

```
initial: a=False and b=False;

P:: a:=!a;

Q:: b:=!b;

R:: await(a) a:=!a;

cobegin P || Q || R coend
```

The transition relation set for the above program is $T = \{t_P, t_Q, t_R\}$ where each transition is executed only once. The program starts at an initial state where a and b are both False. The transition t_P toggles a, the transition t_Q toggles b, and the transition t_R toggles a after a becomes true. Each transition is executed once and then the whole program terminates.

The property we want to check is F(a). (It is in LTL_{-X}, hence it is invariant under stuttering).

(a) List the visible transitions according to this property.

(b) For each pair of transitions state if they are dependent or independent.

(c) Draw the graph for the reachable state space and choose ample sets for each state in the reachable state space (satisfying the constraints C0, C1, C2 and C3 given in the lecture notes).

(d) Construct the corresponding reduced state space using the ample sets computed above.

Hints: 1) While determining the visible transitions choose AP' as $AP' = \{a\}$. 2) Use the following two rules to capture dependent transitions: (a) Two transitions that change the same variable are dependent. (b) If one transition sets a variable and another transition checks that variable, then those two transitions are dependent.

3. Given the following concrete statement "`x := x+1`" and the predicate $x > 0$:

(a) Compute the weakest precondition of "`x := x+1`" with respect to $x > 0$ and compute the weakest precondition of "`x := x+1`" with respect to $\neg(x > 0)$.

(b) Determine if $x > 0$ or $\neg(x > 0)$ imply any of the two preconditions you computed above. Generate the abstraction of the statement "`x := x+1`" with respect to the predicate $x > 0$.