272 – Homework Assignment 2

Fall 2008

Due: Wednesday, October 29th, 5:00PM

Do not discuss the problems with anyone other than the instructor or the TA.

Submission Instructions: Solution to Problem 3 can be submitted electronically or on paper. Solutions to Problems 1 and 2 should be submitted electronically via email to puneet@cs.ucsb.edu with [CS272-HW2-YourLastName] as the subject line.

For problems 1 and 2 your submission should include the Alloy specifications and the output messages from the Alloy Analyzer should be attached as a separate text file.

1. Consider a **Doubly Linked List (DLL)**. If a DLL is not empty, its **head** is a **node** that does not have a **prev** node. If a DLL is not empty, its **tail** is a node that does not have a **next** node. An empty DLL does not have a head node. The **contents** of a DLL is the set of nodes that are reachable from the head node by following the next links, plus the head node. The tail node is included in the contents. For all nodes that are reachable from the head node, the next of the prev of the node is itself.

Write the DLL specification in Alloy. Check or simulate the following properties for scopes 2 and 3 using the Alloy Analyzer: 1) There exist DLLs with 0, 1, 2, and 3 nodes. 2) A DLL does not have a tail node if and only if it is empty. 3) For all DLLs, if head and tail are the same node, then the size of the DLL is 1. 4) No node in a DLL is the prev of two different nodes or the next of two different nodes. 5) There are no cycles (i.e., a node is not reachable from itself by just following next links or by just following prev links).

2. Extend the above DLL specification in Alloy by writing the predicates for the **add** and **delete** operations. Assume that the add operation adds the new node to the head of the DLL (i.e., the new node becomes the new head) and the delete operation removes the tail node (i.e., the prev of the tail becomes the new tail).

Hint: Specify prev and next as relations (using the cross product `->`).

Check or simulate the following properties for scopes 2 and 3 using the Alloy Analyzer: 1) It is possible to obtain an empty DLL after a delete. 2) After an add the DLL contains at least one node. 3) Adding a node to a DLL increases the size of its contents by one. 4) Deleting a node from a DLL decreases the size of its contents by one. 5) After an add, the next of the new head is the old head. 6) After a delete, the new tail is the prev of the old tail.

3. Given the following Alloy formula:

```
r: X ->one Y
all x:X | x != x.r.~r
```

Assume that the scope for X and Y are 2 and generate a SAT formula which is satisfiable only if the above formula is not valid (i.e., negation of the above formula is satisfiable).