#### Automata-based Model Counting



#### Model Counting String Constraint Solver



Aydin et al., Automata-based Model Counting for String Constraints. (CAV'15) USB 2

### Automata Based Counter (ABC) A Model Counting String Constraint Solver



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## String Constraint Language

 $C \longrightarrow bterm$ 

bterm → v | true | false | ¬bterm | bterm ∧ bterm | bterm ∨ bterm | (bterm) | sterm = sterm | match(sterm, sterm) | contains(sterm, sterm) | begins(sterm, sterm) | ends(sterm, sterm) | iterm = iterm | iterm < iterm | iterm > iterm

```
iterm → v | n
| iterm + iterm | iterm - iterm | iterm × n | (iterm)
| length(sterm) | toint(sterm)
| indexof(sterm, sterm)
| lastindexof(sterm, sterm)
```



#### ABC: Constraint language

• A more compact notation

$$\varphi \longrightarrow \varphi \land \varphi \mid \varphi \lor \varphi \mid \neg \varphi \mid \varphi_{\mathbb{Z}} \mid \varphi_{\mathbb{S}} \mid \top \mid \bot$$

$$\varphi_{\mathbb{Z}} \quad \longrightarrow \quad \beta = \beta \mid \beta < \beta \mid \beta > \beta$$

- $\varphi_{\mathbb{S}} \quad \longrightarrow \quad \gamma = \gamma \mid \gamma < \gamma \mid \gamma > \gamma \mid \text{match}(\gamma, \rho) \mid \text{contains}(\gamma, \gamma) \mid \text{begins}(\gamma, \gamma) \mid \text{ends}(\gamma, \gamma)$
- $\beta \longrightarrow \mathbf{v}_i \mid \mathbf{n} \mid \beta + \beta \mid \beta \beta \mid \beta \times \mathbf{n} \\ \mid \text{length}(\gamma) \mid \text{toint}(\gamma) \mid \text{indexof}(\gamma, \gamma) \mid \text{lastindexof}(\gamma, \gamma)$
- $\begin{array}{ll} \gamma & \longrightarrow v_{s} \mid \rho \mid \gamma \cdot \gamma \mid \operatorname{reverse}(\gamma) \mid \operatorname{tostring}(\beta) \mid \operatorname{charat}(\gamma, \beta) \mid \operatorname{toupper}(\gamma) \mid \operatorname{tolower}(\gamma) \\ \mid & \operatorname{substring}(\gamma, \beta, \beta) \mid \operatorname{replacefirst}(\gamma, \gamma, \gamma) \mid \operatorname{replacelast}(\gamma, \gamma, \gamma) \mid \operatorname{replaceall}(\gamma, \gamma, \gamma) \end{array}$

$$\rho \quad \longrightarrow \quad \varepsilon \mid \boldsymbol{s} \mid \rho \cdot \rho \mid \rho \mid \rho \mid \rho^*$$

# Example String Expressions

	String Expression	Constraint Language
Java	s.length()	length(s)
	s.isEmpty()	length(s) == 0
	s.startsWith(t,n)	$0 \le n \land n \le  s  \land$ begins(substring(s,n, s ),t)
	s.indexOf(t,n)	<pre>indexof(substring(s,n, s ),t)</pre>
	s.replaceAll(p,r)	replaceall(s,p,r)
РНР	strrpos(s, t)	lastindexof(s,t)
	<pre>substr_replace(s, t,i,j)</pre>	<pre>substring(s,0,i).t.substring(s,j, s )</pre>
	<pre>strip_tags(s)</pre>	replaceall(s,(" <a>" "" ),"")</a>
	<pre>mysql_real_escape _string(s)</pre>	<pre>replaceall(s     ,replaceall(s,"\\","\\\\")     ,"'", "\'")</pre>



#### Model Counting String Constraint Solver



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#### ABC in a nutshell

Automata-based constraint solving

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Why?

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#### ABC in a nutshell

Automata-based constraint solving

#### **Basic idea:**

Constructing an automaton for the set of solutions of a constraint reduces model counting problem to path counting!

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Automata-based constraint solving

Generate automaton that accepts satisfying solutions for the constraint



Automata-based constraint solving: expr, ¬

Basic string constraints are directly mapped to automata

$$v = ab''$$
 match(v, (ab)\*)  $\neg$ match(v, (ab)\*







automata complement Automata-based constraint solving: expr,  $\neg$ ,  $\land$ ,  $\lor$ 

More complex constraints are solved by creating automata for subformulae then combining their results

> -match(v, (ab)\*)  $\Lambda$  length(v) = 2  $\downarrow^{v}$   $\downarrow^{b}$   $\downarrow^{a,b}$   $\downarrow^{v}$   $\downarrow^{0}$   $\downarrow^{a,b}$   $\downarrow^{a,b}$   $\downarrow^{2}$

> > automata product

#### Automata-based constraint solving: expr, $\neg$ , $\land$ , $\lor$

More complex constraints are solved by creating automata for subformulae then combining their results

 $\neg$ match(v, (ab)\*)  $\land$  length(v) = 2



automata product

#### String Automata Construction: More Details































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 $C \equiv \neg (x \in (01)^*) \land LEN(x) = 2$ 



00, 10, 11



## Relational constraints

- Relational constraints:
  - Constraints that involve multiple variables
- How do we handle relational constraints with automata?


#### Automata-based constraint solving: relational

For multi-variable constraints, generate an automaton for each variable



Automata-based constraint solving: relational

For multi-variable constraints, generate an automaton for each variable



Automata-based constraint solving: relational

Single track automata cannot precisely capture relational constraints

Generated automata significantly over-approximate # of satisfying solutions

Use multi-track automata

#### Multi-track automata

Multi-track automaton = DFA accepting tuples of strings

#### Each track represents the values of a single variable



Preserves relations among variables!

#### Multi-track automata

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Padding symbol  $\lambda \notin \Sigma$  used to align tracks of different length (appears at the end)

Correctly encodes the constraint

# Relational String Constraints: Summary

- How to handle constraints with multiple string variables?
- One approach is to use multiple single-track DFAs
  - One DFA per variable
- Alternative approach: Use one multi-track DFAs
  - Each track represents the values of one string variable
- Using multi-track DFAs:
  - Identifies the relations among string variables
  - Improves the precision
  - Can be used to represent properties that depend on relations among string variables, e.g., \$file = \$usr.txt



### Multi-track Automata

- Let X (the first track), Y (the second track), be two string variables
- λ is the padding symbol
- A multi-track automaton that encodes the word equation:





# Alignment

- To conduct relational string analysis, we need to compute union or intersection of multi-track automata
  - Intersection is closed under aligned multi-track automata
    - In an aligned multi-track automaton λs are right justified in all tracks, e.g., abλλ instead of aλbλ
- However, there exist unaligned multi-track automata that are not equivalent to any aligned multi-track automata
  - Use an alignment algorithm that constructs aligned automata which over or under approximates unaligned ones
    - Over approximation: Generates an aligned multi-track automaton that accepts a super set of the language recognized by the unaligned multitrack automaton
    - Under approximation: Generates an aligned multi-track automaton that accepts a subset of the language recognized by the unaligned multi-track automaton



# Word Equations

- Word equations: Equality of two expressions that consist of concatenation of a set of variables and constants
  - Example: X = Y . txt
- Word equations and their combinations (using Boolean connectives) can be expressed using only equations of the form X = Y . c, X = c .
   Y, c = X .Y, X = Y. Z, Boolean connectives and existential quantification
- Construct multi-track automata from basic word equations
  - The automata should accept tuples of strings that satisfy the equation
- Boolean connectives can be handled using intersection, union and complement
- Existential quantification can be handled using projection



### Word Equations to Automata

- Basic equations X = Y . c, X = c . Y, c = X . Y and their
   Boolean combinations can be represented precisely using multi-track automata
- The size of the aligned multi-track automaton for X = c .
   Y is exponential in the length of c
- The nonlinear equation X = Y.Z cannot be represented precisely using an aligned multi-track automaton



### Word Equations to Automata

- When we cannot represent an equation precisely, we can generate an over or under-approximation of it
  - Over-approximation: The automaton accepts all string tuples that satisfy the equation and possibly more
  - Under-approximation: The automaton accepts only the string tuples that satify the equation but possibly not all of them
- We can implement a function CONSTRUCT(equation, sign)
  - Which takes a word equation and a sign and creates a multi-track automata that over or under-approximation of the equation based on the input sign



### Integer Constraints

 $C \longrightarrow bterm$ 

bterm → v | true | false | ¬bterm | bterm ∧ bterm | bterm ∨ bterm | (bterm) | sterm = sterm | match(sterm, sterm) | contains(sterm, sterm) | begins(sterm, sterm) | ends(sterm, sterm) | iterm = iterm | iterm < iterm | iterm > iterm

iterm → v | n | iterm + iterm | iterm - iterm | iterm × n | (iterm) | length(sterm) | toint(sterm) | indexof(sterm, sterm) | lastindexof(sterm, sterm)



Multi-track automata can also represent Presburger (linear arithmetic) arithmetic constraints

• Each track represents a single numeric variable



# Linear Arithmetic Constraints

- Can be used to represent sets of valuations of unbounded integers
- Linear integer arithmetic formulas can be stored as a set of polyhedra

$$F = \bigvee_{k} \bigwedge_{l} C_{kl}$$

where each  $c_{kl}$  is a linear equality or inequality constraint and each

$$\bigwedge_{l} C_{kl}$$

is a polyhedron

### Automata Representation for Arithmetic Constraints

[Bartzis, Bultan CIAA' 02, IJFCS ' 02]

Given an atomic linear arithmetic constraint in one of the following two forms

$$\sum_{i=1}^{r} a_i \cdot \chi_i = C \qquad \qquad \sum_{i=1}^{r} a_i \cdot \chi_i < C$$

we construct a DFA which accepts all the solutions to the given constraint

 By combining such automata one can handle full Presburger arithmetic (linear arithmetic constraints + quantification)



### **Basic Construction**

- We first construct a basic state machine which
  - Reads one bit of each variable at each step, starting from the least significant bits
  - and executes bitwise binary addition and stores the carry in each step in its state



Automaton Construction

- Equality With 0
  - All transitions writing I go to a sink state
  - State labeled 0 is the only accepting state
  - For disequations  $(\neq)$ , state labeled 0 is the only rejecting state
- Inequality (<0)</p>
  - States with negative carries are accepting
  - No sink state
- Non-zero Constant Term c
  - Same as before, but now -c is the initial state
  - If there is no such state, create one (and possibly some intermediate states which can increase the size by |c|)



Conjunction and Disjunction

Conjunction and disjunction is handled by generating the product automaton

 011
 011
 01

 011
 011
 01



 $C \equiv x = -1 \land x + y = 1$ 









 Using automata construction techniques described in:
 C. Bartzis and Tevfik Bultan. Efficient symbolic representations for arithmetic constraints in verification. Int. J. Found. Comput. Sci., 2003

 $C \equiv x = -1 \land x + y = 1$ 



(|||, 0|0) = (-|, 2)

 Conjunction and disjunction is handled by automata product, negation is handled by automata complement Constraint Solving: Example Combining String and Integer Constraints

 $i = 2 \times j \wedge length(v) = i \wedge match(v, (a | b))$ 



automaton for numeric variables (v<sub>l</sub> auxiliary variable encoding length of v) automaton for string variables

### Apporoximation

- In general ABC constructs automata that over approximate the solution set
  - Some string constraints and combinations of string and integer constraints can lead to non-regular sets,
  - which means they are not representable as automata
- ABC provides a sound over-approximation/abstraction:
  - If the automata does not accept any strings then the original formula is guaranteed to be NOT satisfiable
- It is possible to also provide a sound underapproximation using automata



#### Model Counting String Constraints Solver



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### Can you solve it Will Hunting?

Gas the graph / Find 1) the adjaconay matrix A 2) the matrix giving the number of 3 step walks 3) the generating function for walks from point 2->1 4) the generating function for walks from points 1->3



# Automata-based Model Counting

 Converting constraints to automata reduces the model counting problem to path counting problem in graphs



- We will generate a function f(k)
  - Given length bound k, it will count the number of paths with length k.
  - $f(0) = 0, \{\}$
  - $f(1) = 2, \{0,1\}$
  - $f(2) = 3, \{00, 10, 11\}$

#### Path Counting via Matrix Exponentiation

 $\mathcal{C} = \neg(x \in (01)^*)$ 





### Path Counting via Recurrence Relation

m0 m1 n ... mk

$$f(n,k) = \sum_{(m,n)\in E} f(m,k-1)$$

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f(0,0) = 1f(1,0) = 0f(2,0) = 0

f(i,0) = 0

#### Path Counting via Recurrence Relation

$$f(4,k) = f(2,k-1) + f(3,k-1)$$

$$f(4,k) = f(1,k-1) + f(2,k-1) + f(3,k-1)$$

$$f(1,k) = f(1,k-1)$$

$$f(1,k) = f(2,k-1)$$

$$f(1,0) = 1, f(2,0) = 0, f(3,0) = 0, f(4,0) = 0$$

D



### Path Counting via Recurrence Relation

We can solve system of recurrence relations for final node



$$f(0) = 0, f(1) = 2, f(2) = 3$$
  
$$f(k) = 2f(k-1) + f(k-2) - 2f(k-3)$$

Counting Paths via Generating Functions

We can compute a generating function, g(z), for a DFA from the associated matrix



$$g(z) = (-1)^n \frac{\det(I - zT; n + 1, 1)}{z \times \det(I - zT)} = \frac{2z - z^2}{1 - 2z - z^2 + 2z^3}$$



Counting Paths via Generating Functions

$$g(z) = \frac{2z - z^2}{1 - 2z - z^2 + 2z^3}$$

• Each f(i) can be computed by Taylor expansion of g(z)

$$g(z) = \frac{g(0)}{0!} z^0 + \frac{g^{(1)}(0)}{1!} z^1 + \frac{g^{(2)}(0)}{2!} z^2 + \dots + \frac{g^{(n)}(0)}{n!} z^n + \dots$$
  

$$g(z) = 0z^0 + 2z^1 + 3z^2 + 8z^3 + 15z^4 + \dots$$
  

$$g(z) = f(0)z^0 + f(1)z^1 + f(2)z^2 + f(3)z^3 + f(4)z^4 + \dots$$

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# Good job Will Hunting!

G is the graph 23 Find: I The adjacency matrix, A. 2 The matrix giving the number of 3 step walks 3) The generating function for walks from  $dot(1_i, -ZA_i)$ " The generating function for walks form 1-73 This is correct. 723-222+425) Who did this ? 142 + 18 25 942



Applicable to Both Automata

Multi-track Binary Integer Automaton:



String Automaton:





0

### Model Counting String Constraints Solver



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Automata-based model counting extensions

- In order to scale the automata-based model counting, it is necessary to cache the prior results
- Many constraints generated from programs are equivalent
  - By normalizing constraints we can identify many equivalent constraints
- 87X improvement for the Kaluza big data set

## Kaluza Dataset: 1,342 big constraints and 17,554 small

253		42	42		40		40		
		40	39		38		38		
		39	38		36	36		35	
99	12	39	38		34	28	3	27	
99	43	39	37		32	2	7	13 67	

1,342 big constraints are reduced to 34 equivalent constraints after normalization

2543	1875			1874				1020	
	736	399		345	5 323		216		
2537	1	374	L	195	15	2			
	729			100	94	Ĺ			Ш
		374		186	74	H	H	₩	
	445	371		168 155	73 72 57				

17,554 small constraints are reduced to 360 equivalent constraints after normalization

## Automata-based model counting

extensions

- More caching
  - Cache subformulas
  - Automata provide a canonical form for constraints after minimization and determinization
  - Generate keys for automata and use a compute cache like BDDs
- Subformula caching leads to order of magnitude improvement for attack synthesis

## **ABC DEMO**

https://github.com/vlab-cs-ucsb/ABC

http://ec2-52-35-130-176.us-west-

2.compute.amazonaws.com/