## Automata-based Model Counting

## Model Counting String Constraint Solver

## INPUT

## OUTPUT



Aydin et al.,Automata-based Model Counting for String Constraints. (CAV'I5)

## Automata Based Counter (ABC) A Model Counting String Constraint Solver



Aydin et al.,Automata-based Model Counting for String Constraints. (CAV'I5)

## String Constraint Language

| $C$ | $\longrightarrow$ bterm |
| :---: | :---: |
| bterm | $\longrightarrow \mathrm{v}$ \| true | false |
|  | $\mid \neg$ bterm \| bterm $\wedge$ bterm \| bterm $\vee$ bterm $\mid$ (bterm) |
|  | \| sterm $=$ sterm |
|  | \| match(sterm, sterm) |
|  | contains(sterm, sterm) |
|  | begins(sterm, sterm) |
|  | ends(sterm, sterm) |
|  | $\mid$ iterm $=$ iterm $\mid$ iterm $<$ iterm $\mid$ iterm $>$ iterm |
| iterm | $\longrightarrow \mathrm{V} \mid \mathrm{n}$ |
|  | iterm + iterm $\mid$ iterm - iterm $\mid$ iterm $\times \mathrm{n} \mid$ (iterm) |
|  | \| length(sterm) | toint(sterm) |
|  | \| indexof(sterm, sterm) |
|  | \| lastindexof(sterm, sterm) |
| sterm | $\longrightarrow \mathrm{v}\|\varepsilon\| \mathrm{s}$ |
|  | \| sterm.sterm | sterm|sterm | sterm ${ }^{*}$ \| (sterm) |
|  | \| charat(sterm, iterm) | tostring(iterm) |
|  | \| toupper(sterm) | tolower(sterm) |
|  | substring(sterm, iterm, iterm) |
|  | \| replacefirst(sterm, sterm, sterm) |
|  | \| replacelast(sterm, sterm, sterm) |
|  | \| replaceall(sterm, sterm, sterm) |

## ABC: Constraint language

- A more compact notation

$$
\begin{aligned}
& \varphi \quad \longrightarrow \varphi \wedge \varphi|\varphi \vee \varphi| \neg \varphi\left|\varphi_{\mathbb{Z}}\right| \varphi_{\mathbb{S}}|\top| \perp \\
& \varphi_{\mathbb{Z}} \longrightarrow \beta=\beta|\beta<\beta| \beta>\beta \\
& \varphi_{\mathbb{S}} \longrightarrow \gamma=\gamma|\gamma<\gamma| \gamma>\gamma|\operatorname{match}(\gamma, \rho)| \operatorname{contains}(\gamma, \gamma)|\operatorname{begins}(\gamma, \gamma)| \operatorname{ends}(\gamma, \gamma) \\
& \beta \quad \longrightarrow \quad v_{i}|n| \beta+\beta|\beta-\beta| \beta \times n \\
& |\operatorname{length}(\gamma)| \operatorname{toint}(\gamma)|\operatorname{indexof}(\gamma, \gamma)| \text { lastindexof( } \gamma, \gamma) \\
& \gamma \quad \longrightarrow \quad v_{s}|\rho| \gamma \cdot \gamma \mid \text { reverse }(\gamma) \mid \text { tostring }(\beta)|\operatorname{charat}(\gamma, \beta)| \text { toupper }(\gamma) \mid \text { tolower }(\gamma) \\
& \rho \quad \longrightarrow \varepsilon|s| \rho \cdot \rho|\rho| \rho \mid \rho^{*}
\end{aligned}
$$

## Example String Expressions



## Model Counting String Constraint Solver



Aydin et al.,Automata-based Model Counting for String Constraints. (CAV'I5)

## ABC in a nutshell

Automata-based constraint solving
Why?

## ABC in a nutshell

Automata-based constraint solving

## Basic idea:

Constructing an automaton for the set of solutions of a constraint reduces model counting problem to path counting!

## Automata-based constraint solving

Generate automaton that accepts satisfying solutions for the constraint
$A B C$ can handle both
string and integer constraints

Constraints over only string
variables
(e.g., v = "abcd")

Constraints over only integer
variables
(e.g., $i=2 \times j$ )

Constraints over both string and integer variables
(e.g., length $(v)=i$ )

## Automata-based constraint solving: expr, $\neg$

Basic string constraints are directly mapped to automata

$$
\mathrm{v}=\mathrm{"ab} \text { " }
$$

$$
\operatorname{match}(\mathrm{v},(\mathrm{ab}) \star) \quad \neg \operatorname{match}(\mathrm{v},(\mathrm{ab}) \star)
$$


automata complement

## Automata-based constraint solving: expr, $\neg, \wedge, \vee$

More complex constraints are solved by creating automata for subformulae then combining their results

$$
\neg \operatorname{match}(\mathrm{v},(\mathrm{ab}) *) \wedge \text { length }(\mathrm{v})=2
$$


automata product

## Automata-based constraint solving: expr, $\neg, \wedge, \vee$

More complex constraints are solved by creating automata for subformulae then combining their results

$$
\neg \text { match }(\mathrm{v},(\mathrm{ab}) *) \wedge \text { length }(\mathrm{v})=2
$$


automata product

## String Automata Construction:

 More Details$$
C \equiv \neg\left(x \in(01)^{*}\right) \wedge \operatorname{LEN}(x)=2
$$



## String Automata Construction

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$$


$00,10,11$

## Relational constraints

- Relational constraints:
* Constraints that involve multiple variables
- How do we handle relational constraints with automata?


## Automata-based constraint solving: relational

For multi-variable constraints, generate an automaton for each variable

$$
v=t \quad v \neq t \quad v=t \wedge v \neq t
$$



## Automata-based constraint solving: relational

For multi-variable constraints, generate an automaton for each variable


## Automata-based constraint solving: relational

Single track automata cannot precisely capture relational constraints

Generated automata significantly over-approximate \# of satisfying solutions

## Use multi-track automata

## Multi-track automata

Multi-track automaton $=$ DFA accepting tuples of strings
Each track represents the values of a single variable


Preserves relations among variables!

## Multi-track automata

$$
v=t
$$

$$
v \neq t
$$



Padding symbol $\lambda \notin \Sigma$ used to align tracks of different length (appears at the end)

$$
v=t \wedge v \neq t
$$



Correctly encodes the constraint

## Relational String Constraints: Summary

- How to handle constraints with multiple string variables?
- One approach is to use multiple single-track DFAs
, One DFA per variable
- Alternative approach: Use one multi-track DFAs
- Each track represents the values of one string variable
- Using multi-track DFAs:
- Identifies the relations among string variables
- Improves the precision
- Can be used to represent properties that depend on relations among string variables, e.g., \$file = \$usr.txt


## Multi-track Automata

- Let X (the first track), Y (the second track), be two string variables
- $\lambda$ is the padding symbol
- A multi-track automaton that encodes the word equation:



## Alignment

- To conduct relational string analysis, we need to compute union or intersection of multi-track automata
- Intersection is closed under aligned multi-track automata
- In an aligned multi-track automaton $\lambda$ s are right justified in all tracks, e.g., $a b \lambda \lambda$ instead of $a \lambda b \lambda$
- However, there exist unaligned multi-track automata that are not equivalent to any aligned multi-track automata
- Use an alignment algorithm that constructs aligned automata which over or under approximates unaligned ones
- Over approximation: Generates an aligned multi-track automaton that accepts a super set of the language recognized by the unaligned multitrack automaton
- Under approximation: Generates an aligned multi-track automaton that accepts a subset of the language recognized by the unaligned multi-track automaton


## Word Equations

- Word equations: Equality of two expressions that consist of concatenation of a set of variables and constants
- Example: $\mathrm{X}=\mathrm{Y} . \mathrm{txt}$
- Word equations and their combinations (using Boolean connectives) can be expressed using only equations of the form $X=Y . c, X=c$. $\mathrm{Y}, \mathrm{c}=\mathrm{X} . \mathrm{Y}, \mathrm{X}=\mathrm{Y} . \mathrm{Z}$, Boolean connectives and existential quantification
- Construct multi-track automata from basic word equations
- The automata should accept tuples of strings that satisfy the equation
- Boolean connectives can be handled using intersection, union and complement
- Existential quantification can be handled using projection


## Word Equations to Automata

- Basic equations $X=Y$. $c, X=c . Y, c=X . Y$ and their Boolean combinations can be represented precisely using multi-track automata
- The size of the aligned multi-track automaton for $\mathrm{X}=\mathrm{c}$. $Y$ is exponential in the length of $c$
- The nonlinear equation $X=Y$. $Z$ cannot be represented precisely using an aligned multi-track automaton


## Word Equations to Automata

- When we cannot represent an equation precisely, we can generate an over or under-approximation of it
* Over-approximation:The automaton accepts all string tuples that satisfy the equation and possibly more
- Under-approximation:The automaton accepts only the string tuples that satify the equation but possibly not all of them
, We can implement a function CONSTRUCT(equation, sign)
- Which takes a word equation and a sign and creates a multi-track automata that over or under-approximation of the equation based on the input sign


## Integer Constraints

C

$$
\longrightarrow \text { bterm }
$$

bterm

iterm

sterm

```
\longrightarrow
```

    sterm.sterm | sterm|sterm | sterm \({ }^{*}\) | (sterm)
    charat(sterm, iterm) | tostring(iterm)
    toupper(sterm) | tolower(sterm)
    substring(sterm, iterm, iterm)
    replacefirst(sterm, sterm, sterm)
    replacelast(sterm, sterm, sterm)
    replaceall(sterm, sterm, sterm)
    
## Multi-track automata

Multi-track automata can also represent Presburger (linear arithmetic) arithmetic constraints

- Each track represents a single numeric variable
- Encoded as binary integers in 2's complement form

$$
i=j
$$


$i \neq j$
$i=2 \times j$



## Linear Arithmetic Constraints

- Can be used to represent sets of valuations of unbounded integers
- Linear integer arithmetic formulas can be stored as a set of polyhedra

$$
F=\vee_{k} \wedge_{l} c_{k l}
$$

where each $c_{k l}$ is a linear equality or inequality constraint and each

$$
\wedge_{l} c_{k l}
$$

is a polyhedron

## Automata Representation for Arithmetic Constraints <br> [Bartzis, Bultan CIAA' 02, IJFCS ' 02]

- Given an atomic linear arithmetic constraint in one of the following two forms

$$
\sum_{i=1}^{v} a_{i} \cdot x_{i}=c
$$

$$
\sum_{i=1}^{v} a_{i} \cdot x_{i}<c
$$

we construct a DFA which accepts all the solutions to the given constraint

- By combining such automata one can handle full Presburger arithmetic (linear arithmetic constraints + quantification)


## Basic Construction

- We first construct a basic state machine which
- Reads one bit of each variable at each step, starting from the least significant bits
v and executes bitwise binary addition and stores the carry in each step in its state

Example
$x+2 y$

010
$+2 \times 001$

100


Number of states: $O\left(\sum_{i=1}^{v}\left|a_{i}\right|\right)$

## Automaton Construction

- Equality With 0
- All transitions writing I go to a sink state
- State labeled 0 is the only accepting state
- For disequations $(\neq)$, state labeled 0 is the only rejecting state
- Inequality (<0)
- States with negative carries are accepting
- No sink state
- Non-zero Constant Term c
- Same as before, but now -c is the initial state
- If there is no such state, create one (and possibly some intermediate states which can increase the size by $|\mathrm{c}|$ )


## Conjunction and Disjunction

- Conjunction and disjunction is handled by generating the product automaton

| 001 | 01 | 1 |
| :--- | :--- | :--- |
| $0,1,1$ | 0,1 | 0 |

Automaton for $x-y<1$

Automaton for $2 x-y>0$


## Integer Automata Construction

$$
C \equiv x=-1 \wedge \mathrm{x}+\mathrm{y}=1
$$



## Integer Automata Construction

$$
\begin{gathered}
C \equiv x=-1 \wedge \mathrm{x}+\mathrm{y}=1 \\
C_{1} \equiv x+0 * y+1=0 \Rightarrow\left[\begin{array}{ll}
1 & 0
\end{array}\right] \\
C_{2} \equiv x+y-1=0 \Rightarrow\left[\begin{array}{lll}
1 & 1 & -1
\end{array}\right]
\end{gathered}
$$



## Integer Automata Construction

$$
\begin{gathered}
C \equiv x=-1 \wedge \mathrm{x}+\mathrm{y}=1 \\
C_{1} \equiv x+0 * y+1=0 \Rightarrow\left[\begin{array}{lll}
1 & 0 & 1
\end{array}\right] \\
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1 & 1 & -1
\end{array}\right]
\end{gathered}
$$



- Using automata construction techniques described in:
C. Bartzis and Tevfik Bultan. Efficient symbolic representations for arithmetic constraints in verification. Int.J. Found. Comput. Sci., 2003


## Integer Automata Construction

$$
C \equiv x=-1 \wedge \mathrm{x}+\mathrm{y}=1
$$


$(111,010)=(-1,2)$

- Conjunction and disjunction is handled by automata product, negation is handled by automata complement


## Constraint Solving: Example Combining String

 and Integer Constraints$$
i=2 \times j \wedge \text { length }(v)=i \wedge \operatorname{match}(v,(a \mid b) *)
$$


automaton for numeric variables
automaton for string variables

## Apporoximation

- In general ABC constructs automata that over approximate the solution set
- Some string constraints and combinations of string and integer constraints can lead to non-regular sets,
- which means they are not representable as automata
- ABC provides a sound over-approximation/abstraction:
- If the automata does not accept any strings then the original formula is guaranteed to be NOT satisfiable
- It is possible to also provide a sound underapproximation using automata


## Model Counting String Constraints Solver

INPUT
string

constraint: $\rightarrow$\begin{tabular}{c}
Automata-Based <br>

| model Counting |
| :--- |
| string constraint |
| solver |
| (ABC) | <br>

OUTPUT <br>
function:
\end{tabular}

Aydin et al.,Automata-based Model Counting for String Constraints. (CAV'I5)

## Can you solve it Will Hunting?



## Automata-based Model Counting

- Converting constraints to automata reduces the model counting problem to path counting problem in graphs

- We will generate a function $f(k)$
- Given length bound $k$, it will count the number of paths with length $k$.
- $f(0)=0,\{ \}$
- $f(1)=2,\{0,1\}$
- $f(2)=3,\{00,10,11\}$


## Path Counting via Matrix Exponentiation

$$
C=\neg\left(x \in(01)^{*}\right)
$$



$$
\begin{array}{r}
T=\left[\begin{array}{llll}
0 & 1 & 0 & 01 \\
1 & 0 & 1 & 1 \\
0 & 0 & 2 & 1 \\
0 & 0 & 0 & 0
\end{array}\right], T^{2}=\left[\begin{array}{llll}
1 & 0 & 2 & 27 \\
0 & 1 & 3 & 1 \\
0 & 0 & 4 & 2 \\
0 & 0 & 0 & 0
\end{array}\right], T^{3}=\left[\begin{array}{llll}
0 & 1 & 3 & 37 \\
1 & 0 & 7 & 4 \\
0 & 0 & 8 & 4 \\
0 & 0 & 0 & 0
\end{array}\right], T^{4}=\left[\begin{array}{llll}
0 & 1 & 1: 87 \\
1 & 0 & 1 & 7 \\
0 & 0 & 7 & 7 \\
0 & 0 & 0 & 8 \\
0 & 0 & 0
\end{array}\right] \\
\\
f(0)=0 \quad f(1)=2 \quad f(2)=3 \quad f(3)=8
\end{array}
$$

## Path Counting via Recurrence Relation



$$
f(n, k)=\sum_{(m, n) \in E} f(m, k-1)
$$

$$
f(0,0)=1
$$

$$
f(1,0)=0
$$

$$
f(2,0)=0
$$

$$
f(i, 0)=0
$$

## Path Counting via Recurrence Relation



## Path Counting via Recurrence Relation

- We can solve system of recurrence relations for final node


$$
\begin{aligned}
& f(0)=0, f(1)=2, f(2)=3 \\
& f(k)=2 f(k-1)+f(k-2)-2 f(k-3)
\end{aligned}
$$

## Counting Paths via Generating Functions

- We can compute a generating function, $g(z)$, for a DFA from the associated matrix


$$
T=\left[\begin{array}{llll}
0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 \\
0 & 0 & 2 & 1 \\
0 & 0 & 0 & 0
\end{array}\right]
$$

$$
g(z)=(-1)^{n} \frac{\operatorname{det}(I-z T: n+1,1)}{z \times \operatorname{det}(I-z T)}=\frac{2 z-z^{2}}{1-2 z-z^{2}+2 z^{3}}
$$

## Counting Paths via Generating Functions

$$
g(z)=\frac{2 z-z^{2}}{1-2 z-z^{2}+2 z^{3}}
$$

- Each $f(i)$ can be computed by Taylor expansion of $g(z)$

$$
\begin{aligned}
& g(z)=\frac{g(0)}{0!} z^{0}+\frac{g^{(1)}(0)}{1!} z^{1}+\frac{g^{(2)}(0)}{2!} z^{2}+\cdots+\frac{g^{(n)}(0)}{n!} z^{n}+\cdots \\
& g(z)=0 z^{0}+2 z^{1}+3 z^{2}+8 z^{3}+15 z^{4}+\cdots \\
& g(z)=f(0) z^{0}+f(1) z^{1}+f(2) z^{2}+f(3) z^{3}+f(4) z^{4}+\cdots
\end{aligned}
$$

## Good job Will Hunting!

$G$ is the graph


Foul
1 The adjacency matrix, A.
2 The mathis pining the number of 3 step walks
3) The generating fenctive for milks from $1 \rightarrow$
4) The generating fandiru for walls pome

$$
1 \rightarrow 3
$$

This is correct. Who did this ?


## Applicable to Both Automata

- Multi-track Binary Integer Automaton:

- String Automaton:



## Model Counting String Constraints Solver

## INPUT

string constraint:

C


## OUTPUT

counting
function:
$\boldsymbol{f}_{\boldsymbol{c}} \longleftarrow$ length bound: $\boldsymbol{k}$ $\downarrow$
$\#$ of strings with length $\leq \boldsymbol{k}$ for which $\boldsymbol{C}$ evaluates to true

## Automata-based model counting extensions

- In order to scale the automata-based model counting, it is necessary to cache the prior results
- Many constraints generated from programs are equivalent
- By normalizing constraints we can identify many equivalent constraints
- $87 \times$ improvement for the Kaluza big data set


## Kaluza Dataset:

1,342 big constraints and 17,554 small

| 253 | 42 | 42 |  |  | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 40 | 39 |  |  | 38 |
|  | 39 | 38 | 36 | 36 | 35 |
| 99 | 39 | 38 | 34 | 28 | 27 |
| 99 | 39 | 37 | 32 | 27 | 13 |
|  |  |  |  | 15 | 3 |

I,342 big constraints are reduced to 34 equivalent constraints after normalization

| 2543 | 1875 |  | 1874 |  | 1020 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 736 | 399 | 345 | 323 | 216 |
| 2537 |  | 374 | 1951 |  |  |
|  | 729 | 374 | $186 \frac{94}{74}$ |  |  |
|  | 445 | 371 | $155 \frac{7}{5}$ |  |  |

I7,554 small constraints are reduced to 360 equivalent constraints after normalization

## Automata-based model counting extensions

- More caching
- Cache subformulas
- Automata provide a canonical form for constraints after minimization and determinization
- Generate keys for automata and use a compute cache like BDDs
- Subformula caching leads to order of magnitude improvement for attack synthesis


## ABC DEMO

https://github.com/vlab-cs-ucsb/ABC
http://ee2-52-35-130-176.us-west2.compute.amazonaws.com/

