Sudoku as an Expression in Propositional Logic

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INTRODUCTION

Sudoku is a kind of puzzle that involves a $9 \times 9$ array of squares. You can learn about it from the Wikipedia entry on Sudoku: [http://en.wikipedia.org/wiki/Sudoku](http://en.wikipedia.org/wiki/Sudoku). The puzzle summarily is to fill in each empty square with a nonzero digit such that:

1. Each row contains all the nonzero digits
2. Each column contains all the nonzero digits
3. Each emboldened sub-array (also known as a region) contains all the nonzero digits.

THE PROBLEM

Represent a proposed solution to a Sudoku puzzle as

```java
int solution[][] = new int[9][9];
```

where `solution[i][j] == k` means that the square associated with row `i` and column `j` has digit `k`. Define

```java
boolean v[][][] = new boolean[9][9][9];
```

**Definition:** An assignment to `solution` corresponds to an assignment to `v` when

```java
solution[i][j] == k ⇔ v(i, j, k).
```

THE PROBLEM SPECIFICATION

**Input:** An instance $I$ of a Sudoku puzzle.

**Output:** A propositional expression, $\varepsilon$, over $v$, such that there is an assignment to $v$ that satisfies $\varepsilon$ if and only if $v$ has a corresponding solution that represents a solution to $I$.

**Challenge:** Describe a procedure that, given an input, produces a correct output as described above.