

Sudoku as an Expression in Propositional Logic

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INTRODUCTION

Sudoku is a kind of puzzle that involves a 9×9 array of squares. You can learn about it from the Wikipedia entry on Sudoku: <http://en.wikipedia.org/wiki/Sudoku>. The puzzle summarily is to fill in each empty square with a nonzero digit such that:

1. Each row contains all the nonzero digits
2. Each column contains all the nonzero digits
3. Each emboldened sub-array (also known as a *region*) contains all the nonzero digits.

THE PROBLEM

Represent a proposed solution to a Sudoku puzzle as

```
int solution[][] = new int[9][9];
```

where `solution[i][j] == k` means that the square associated with row i and column j has digit k .
Define

```
boolean v[][][] = new boolean[9][9][9];
```

Definition: An assignment to `solution` corresponds to an assignment to `v` when

$$\text{solution}[i][j] == k \Leftrightarrow v(i, j, k).$$

THE PROBLEM SPECIFICATION

Input: An instance I of a Sudoku puzzle.

Output: A propositional expression, ε , over v , such that there is an assignment to v that satisfies ε if and only if v has a corresponding `solution` that represents a solution to I .

Challenge: Describe a procedure that, given an input, produces a correct output as described above.