1. $k^n$

2. (a) For any $uv$ in $L_2$ we need to show that $(uv)^R = uv$:
   
   $$(uv)^R = v^R u^R = v^R (v^R)^R = v^R v = uv$$

   (b) Find a word $w$ such that $w$ is in $L_1$ but not in $L_2$ for example $w = aba$

3. (a) Always
   (b) if $\lambda \in L$
   (c) if $\lambda \in L$

4. Prove it in 2 steps: first, show that if $x \in (L_1^* L_2^*)^*$ then $x \in (L_1 \cup L_2)^*$. Second show that if $x \in (L_1 \cup L_2)^*$ then $x \in (L_1^* L_2^*)^*$. To prove each part you can use induction. For example for the first part, what we have to do is:
   
   Basis: show that $(L_1^* L_2^*)^0 = \lambda \in (L_1 \cup L_2)^* $

   Inductive hypothesis: assume that $(L_1^* L_2^*)^k \in (L_1 \cup L_2)^*$ for $k = 1, 2, ..., n$

   Inductive Step: show that $(L_1^* L_2^*)^{n+1} \in (L_1 \cup L_2)^*$

5. (a) $(ab)^* a$
   (b) $(ab)^*$
   (c) $\lambda$

6. (a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$

   If $x \in (L_1 \cup L_2)^R \iff x^R \in ((L_1 \cup L_2)^R)^R \iff x^R \in L_1 \cup L_2 \iff x^R \in L_1$ or $x^R \in L_2 \iff x \in L_1^R \cup L_2^R$

   (b) proof by induction or similar to the previous step

7. (a) $S \rightarrow AaA$

   $$A \rightarrow bA \mid \lambda$$

   (b) $S \rightarrow AaA$

   $$A \rightarrow aA \mid bA \mid \lambda$$
(c) \[ S \to BABABAB \]
\[ A \to a \mid \lambda \]
\[ B \to bB \mid \lambda \]

(d) \[ S \to BaBaBaB \]
\[ B \to bB \mid aB \mid \lambda \]

8. We use induction to prove this. Consider \( x \) as the number of occurrences of \( ab \) in the \( w \).

Induction basis: for the first word \( ab \), \( x = 1 \) which is an odd number.

Inductive assumption: for word \( w \) of length \( n \), \( x \) is an odd number.

Inductive step: to increase the length of \( w \) to \( n + 1 \) we need to add a symbol. This symbol can be added in any of the following positions:
- between \( a \) and \( a \),
- between \( a \) and \( b \),
- between \( b \) and \( a \), or
- between \( b \) and \( b \).

Since we have two symbols in our alphabet, we will have eight different combinations.
It is easy to see that \( x \) is odd in each case.

9. We first show that the length of \( x \) is even. We use contradiction. Suppose that \( |x| \) is an odd number, so:
\[ x^2 = wwx \implies 2|x| = |w| + |x| + |w| \implies 2|x| = 2|w| + |x| \]
It is clear that \( 2|x| \) and \( 2|w| \) are both even so \( |x| \) can not be odd.

Next, we split \( x \) in half such that \( x = yz \) and \( |y| = |z| \). So we will have \( yzyz = wyzw \)
Also we know that \( |y| = |z| \), and \( |yz| = |wy| = |zw| \) so it is easy to show that \( |y| = |z| = |w| \) and finally \( y = z = w \), so: \( x = yz = ww = w^2 \)

10. The answer is shown below: