CS 138: Formal languages and Automata
Homework 3 solutions

1. 

![Diagram](image1.png)

![Diagram](image2.png)

2. 

3. (a) $\delta^*(q_0, 1011) = \{q_2\}$
   
   (b) $\delta^*(q_1, 01) = \{q_1\}$

4. 

1
5. Since $L$ is a regular language, we can construct a corresponding DFA, $M$, such that $L(M) = L$. By definition, $L^R$ consists of all strings in language $L$ in reverse order. We will construct an NFA, $M_R$, representing $L^R$ such that $L(M_R) = L^R$. $M_R$ will contain an additional start state with $\lambda$-transitions to the final states of $M$. The direction of every transition in $M$ is reversed. Also, the start state of $M$ will be the final state of $M_R$. The construction of the NFA $M_R$ is as follows:

Let $M = (Q, \Sigma, \delta, q_L, F)$

$M_R = (Q \cup \{q_0\}, \Sigma_r, \delta_r, q_0, \{q_L\})$ and $q_0 \notin Q$

$p \in \delta(q, a) \iff q \in \delta_r(p, a)$ for $a \in \Sigma$

Now we need to show that $w \in L$ iff $w^R \in L^R$

6. Let $L$ be a regular language. This implies that there exists a DFA, $M = (Q, \Sigma, \delta, q_0, F)$ which accepts $L$. We construct a NFA $M'$ which accepts $\text{chopLeft}(L)$. $M'$ has the same set of states, we only need to make the transitions from the initial state to other states as $\lambda$ transition. Note that if the initial state $q_0$ has a loop back transition, we don’t change that transition. Clearly $M'$ accepts $\text{chopLeft}(L)$. Hence $\text{chopLeft}(L)$ is regular.

$\forall \delta(q_0, a_i) = q_j :$ if $q_j \neq q_i \implies \delta(q_0, \lambda) = q_j$

7. (a) $b^*ab^*ab^*$

(b) $b^*(a + \lambda)b^*(a + \lambda)b^*(a + \lambda)b^*$

(c) $(a + b + c)^*a(a + b + c)^*b(a + b + c)^*c(a + b + c)^* + (a + b + c)^*a(a + b + c)^*c(a + b + c)^*b(a + b + c)^* + (a + b + c)^*b(a + b + c)^* + (a + b + c)^*b(a + b + c)^*a(a + b + c)^*c(a + b + c)^* + (a + b + c)^*b(a + b + c)^*$
\[(a + b + c)^*c(a + b + c)^*a(a + b + c)^* + (a + b + c)^*c(a + b + c)^*b(a + b + c)^*
+ (a + b + c)^*c(a + b + c)^*b(a + b + c)^*a(a + b + c)^*\]

8. 

9. (a) \((1 + 0)^*(10)\)
(b) \(\lambda + 0 + 1 + (0 + 1)^*(00 + 01 + 11)\)
(c) \(1^*0(1 + 01^*0)^*\)

10. a) 

b) 

c)