University of California, Santa Barbara

CMPSC 138 FALL 2017

Homework IV: Due Tuesday, October 31, 4:00 pm. in CS 138 HW box in Room 2108 in HAROLD FRANK HALL.

You can also turn in your homework in class, at the end of Lecture on Tuesday.

Instructions:

- Your solution must be stapled. Put your name and email address on the first page.

- Write your solutions clearly, with appropriate mathematical rigor and care. Justify all steps of your solution. Partially incorrect solutions can still be worth several points, but unjustified answers will result in zero points for the corresponding question.

- You are not allowed to copy or transcribe answers to homework assignments from others or other sources.

- You are not allowed to post solutions of your homework on the Piazza Q&A. Moreover, if you use facts from the online discussion, you should provide your own justification in your solution.

- You are allowed to discuss homework assignments with others, but you must write your answers independently. You should always be able to argue and explain your answers when asked for clarifications.

- Please note that there are no late homeworks allowed.

Homework IV problems:

1. Give regular expressions for the following languages over $\Sigma = \{a, b\}$:

   (a) All words that have $ab$ as the first two and $ba$ as the last two letters.

   (b) All words that do not have $ab$ as the first two letters.
2. What is the shortest length string \( w \) over \( \{a, b\} \) which is not in the language denoted by the regular expression \( b^*(abb^*)^*a^* \)?

3. Consider the toy shown in the following figure. A marble is dropped in at \( A \) or \( B \). Levers \( x_1, x_2 \) and \( x_3 \) cause the marble to fall either to the left or to the right. Whenever a marble encounters a lever, it causes the lever to change state, so that the next marble to encounter the lever will take the opposite branch.

(a) Model this toy by a finite automaton. Denote a marble in at \( A \) by a 0-input and a marble in at \( B \) by 1-input. A sequence of input is accepted if the last marble comes out at \( D \).

(b) Describe the language accepted by the automaton.

4. Find a regular expression for the language over \( \{0, 1\} \) accepted by the following automaton:

5. Suppose \( \mathcal{L} \) is the language accepted by the DFA \( M \) pictured (with start state \( q_1 \)) and consider the homomorphism \( f : \{0, 1\}^* \rightarrow \{a, b\}^* \) defined by

\[
f(0) = abb, \quad f(1) = baa.
\]

(a) Construct a DFA that accepts \( f^{-1}(\mathcal{L}) \).

(b) Construct a regular expression that denotes \( f(0^*1 + 01^*0) \).
(c) Construct a regular expression that denotes $f^{-1}(\mathcal{L})$.

6. Decide whether each statement below is true or false. If it is true, prove it. If not, give a counterexample. The alphabet is $\{a, b\}$.

(a) If $\mathcal{L}_1 \subseteq \mathcal{L}_2$ and $\mathcal{L}_1$ is not regular, then $\mathcal{L}_2$ is not regular.
(b) If $\mathcal{L}_1 \subseteq \mathcal{L}_2$ and $\mathcal{L}_2$ is not regular, then $\mathcal{L}_1$ is not regular.
(c) If $\mathcal{L}_1$ and $\mathcal{L}_2$ are nonregular, then $\mathcal{L}_1 \cup \mathcal{L}_2$ is nonregular.
(d) If $\mathcal{L}_1$ and $\mathcal{L}_2$ are nonregular, then $\mathcal{L}_1 \cap \mathcal{L}_2$ is nonregular.
(e) If $\mathcal{L}_1$ is regular and $\mathcal{L}_2$ is nonregular, then $\mathcal{L}_1 \cap \mathcal{L}_2$ is nonregular.
(f) If $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \ldots$ are all regular, then $\bigcup_{n=1}^{\infty} \mathcal{L}_n$ is regular.
(g) If $\mathcal{L}_2$ is regular and $\mathcal{L}_1$ is finite, then $\mathcal{L}_1 \mathcal{L}_2$ is regular.
(h) If $\mathcal{L}_1 \cup \mathcal{L}_2$ is regular and $\mathcal{L}_1$ is finite, then $\mathcal{L}_2$ is regular.
(i) If $\mathcal{L}_1 \mathcal{L}_2$ is regular and $\mathcal{L}_1$ is finite, then $\mathcal{L}_2$ is regular.

7. Define the operation $\text{truncate}$, which removes the rightmost letter from any nonnull string (this is the operation $\text{chopright}$ of the text). For example, $\text{truncate}(aaaba)$ is $aaab$. This operation can be extended to languages not containing $\lambda$ by

$$\text{truncate}(\mathcal{L}) = \{ \text{truncate}(w) \mid w \in \mathcal{L} \}.$$  

Use the closure properties of regular languages to show that for any regular $\mathcal{L}$ not containing $\lambda$, $\text{truncate}(\mathcal{L})$ is regular.

8. Describe an algorithm for determining whether or not a regular language $\mathcal{L}$ contains a string $w$ of even length such that $w^R \in \mathcal{L}$.

9. Use the pumping lemma to show that the language $\mathcal{L} = \{a^m b^k \mid m > k\}$ is nonregular.

10. Show that $\mathcal{L} = \{a^m \mid m \geq 0\}$ is nonregular.