1. Which one(s) of the following languages over $\Sigma = \{a\}$ are regular? You need to justify your answers.

(a) $L_1 = \{a^{2m} \mid m \geq 0\}$,
(b) $L_2 = \{a^{2m} \mid m \geq 0\}$,
(c) $L_1 / L_2$,
(d) $L_1 \cap \{w \in \Sigma^* \mid |w| \leq 2^{1000}\}$,
(e) $L_2 \cap \{w \in \Sigma^* \mid |w| \geq 2^{1000}\}$.

2. Suppose $L$ is the language accepted by the DFA $M$ pictured (with start state $q_1$) and consider the homomorphism $f : \{0,1\}^* \to \{a,b\}^*$ defined by $f(0) = abb$, $f(1) = baa$.

(a) Construct a DFA that accepts $f^{-1}(L)$.
(b) Construct a regular expression that denotes $f(0^*1 + 01^*0)$.
(c) Construct a regular expression that denotes $f^{-1}(L)$.

3. Do Problem 15, Section 4.1 of the text.

4. Decide whether each statement below is true or false. If it is true, prove it. If not, give a counterexample. The alphabet is $\{a,b\}$.

(a) If $L_1 \subseteq L_2$ and $L_1$ is not regular, then $L_2$ is not regular.
(b) If $L_1 \subseteq L_2$ and $L_2$ is not regular, then $L_1$ is not regular.
(c) If $L_1$ and $L_2$ are nonregular, then $L_1 \cup L_2$ is nonregular.
(d) If $L_1$ and $L_2$ are nonregular, then $L_1 \cap L_2$ is nonregular.

(e) If $L_1$ is regular and $L_2$ is nonregular, then $L_1 \cap L_2$ is nonregular.

(f) If $L_1, L_2, L_3, \ldots$ are all regular, then $\cup_{n=1}^{\infty} L_n$ is regular.

(g) If $L_2$ is regular and $L_1$ is finite, then $L_1L_2$ is regular.

(h) If $L_1 \cup L_2$ is regular and $L_1$ is finite, then $L_2$ is regular.

(i) If $L_1L_2$ is regular and $L_1$ is finite, then $L_2$ is regular.

5. Define the operation $\text{truncate}$, which removes the rightmost letter from any nonnull string. For example, $\text{truncate}(aaaba) = aaab$. This operation can be extended to languages not containing $\lambda$ by

$$\text{truncate}(L) = \{ \text{truncate}(w) \mid w \in L \}.$$

Show that for any regular $L$ not containing $\lambda$, $\text{truncate}(L)$ is regular.

6. Describe an algorithm for determining whether or not a regular language $L$ contains a string $w$ of even length such that $w^R \in L$.

7. Use the pumping lemma to show that the language $L = \{a^mb^k \mid m > k \}$ is not regular.

8. Do Problem 19, Section 4.3 of the text.