1. if $L$ is regular, so $\bar{L}$. Since intersection of two regular languages is regular, therefore $L_1 = \bar{L} \cap L(w_1cw_2) = a^n b^n$ has to be regular. However it is easy to show that $L_1$ is not regular using Pumping Lemma.

2. if $L$ is regular, so $L^R$. We can write $L$ as $\{w : w \in L_1, w \in L_2 \}$.

3. (a) It’s false. We can give $L_1 = \{a^n b^n \mid n \geq 0\}$ and $L_2 = \Sigma^*$ as counterexamples. Here $L_1 \subseteq L_2$ and $L_1$ is not regular. But $L_2$ is regular.

(b) It’s false. We can give $L_1 = \{ab\}$ and $L_1 = \{a^n b^n \mid n \geq 0\}$ as counterexamples. Here $L_1 \subseteq L_2$ and $L_2$ is not regular. But $L_2$ is regular.

(c) It’s false. We can give $L_1 = \{a^n b^n \mid n \geq 0\}$ and $L_2 = \Sigma^*$ as counterexamples. Here $L_1 \subseteq L_2$ and $L_2$ are not regular. But $L_1 \cup L_2 = \Sigma^*$ is regular.

(d) It’s false. We can give $L_1 = \{a^n b^n \mid n \geq 0\}$ and $L_2 = \{b^n a^n \mid n \geq 0\}$ as counterexamples. Here $L_1$ and $L_2$ are not regular. But $L_1 \cap L_2 = \lambda$ is regular.

(e) It’s false. We can give $L_1 = \{ab\}$ and $L_2 = \{a^n b^n \mid n \geq 0\}$ as counterexamples. Here $L_1$ is regular and $L_2$ is not regular. But $L_1 \cap L_2 = \{ab\}$ is regular.

(f) It’s false. We can give $L_1 = \{ab\}, L_2 = \{a^n b^n \mid n \geq 0\}$ as counterexamples. Here $L_1, L_2, \ldots, L_n$ are all regular. But $\bigcup_{n=1}^{\infty} L_n = \{a^n b^n \mid n > 0\}$ is nonregular.

(g) It’s true. If $L_1$ is finite then $L_1$ is regular, so both $L_1$ and $L_2$ is a regular language and regular languages are closed under concatenation.

(h) It’s true. We can write $(L_1 \cup L_2) - L_1 = L_2 - L_1$ and $L_2 = (L_1 \cap L_2) \cup (L_2 - L_1)$. Both $L_1$ and $L_1 \cup L_2$ are regular, therefore $L_2 - L_1$ is regular because set difference is closed under regular languages. $L_1 \cap L_2$ is regular because since $L_1$ is finite, the intersection is finite, therefore regular. $L_2$ is union of two regular languages, therefore it is regular.

(i) It’s false. We can give $L_1 = \{\lambda, a\}, L_2 = \{a^n \mid n \geq 4\}$ as counterexamples. In this case, $L_1 L_2$ is regular, $L_1$ is finite but $L_2$ is nonregular.
5. No, here is a grammar that follows the given rules, but generates the non-regular language $a^n b^n$.

$$S \to aA | \lambda$$

$$A \to Sb$$

6. (a) $S \to aS | b$

(b) $S \to Ab$

$$A \to Aa | \lambda$$

7. (a) $a^n b$

(b) $ba^4$

(c) $a^* b + b^+ a$

(d) .

8. Take the DFA $M$, add $\lambda$ transitions from initial state to all other states and $\lambda$ transitions from all non-final states to the final states, the resulting NFA accepts all substrings of $L$ including $\lambda$.

9. Choose $\omega = a^{\lceil n! \rceil} b^{(n+1)!}$, clearly because $|xy| \leq n$, $y = a^t$, $0 < t \leq n$, so $xy^k z = a^{\lceil n! \rceil + (k-1) \lceil (n+1)! \rceil}$ which means if $k = \frac{n! n! - 1}{t} + 1$ then $xy^k z = a^{(n+1)! - 1} b^{(n+1)!}$
10. (a) 
   (b) $a(ab)^*b$
   (c) $S \rightarrow Ab$
       $A \rightarrow Aaba|a$