Homework V: Due Friday, November 9 by 4pm in the CS138 HW box in HFH 2108.

1. Consider the CFG \( G = (\{S, A\}, \{a, b\}, P, S) \) where \( P \) consists of five \( S \)-productions and four \( A \)-productions as given below:

\[
S \rightarrow aaS \mid bbS \mid abA \mid baA \mid \lambda \\
A \rightarrow aaA \mid bbA \mid abS \mid baS.
\]

What is \( L(G) \)? Prove your answer.

2. Construct CFGs for the following languages

(a) \( \{a^i b^i c^i : i \geq 0\} \)

(b) \( \{a^i b^j : 0 \leq i \leq j\} \).

3. Consider the language \( L \) over the alphabet \( \{a, b, c\} \) consisting of all strings of the form \( a^i b^i c^k \), where \( i, j, k > 0 \) and either \( i = j \) or \( j = k \). Is \( L \) regular? Is it context-free? Justify your answers.

4. For each word below, determine whether or not it is generated by each CFG given on the right, and draw a derivation tree for it if it is:

\[
aabb \quad CFG 1. \quad S \rightarrow aSb \mid ab \\
abaa \quad CFG 2. \quad S \rightarrow aS \mid bS \mid a \\
abba \quad CFG 3. \quad S \rightarrow aS \mid aSb \mid X ; \quad X \rightarrow aXa \mid a \\
aaaa \quad CFG 4. \quad S \rightarrow aAS \mid a ; \quad A \rightarrow SbA \mid SbA \\
\]

5. Consider the CFG \( G \)

\[
S \rightarrow aSA \mid \lambda \\
A \rightarrow aA \mid a
\]

(a) Show that \( G \) is ambiguous.

(b) Find an unambiguous CFG equivalent to \( G \).

(c) Find an unambiguous CFG that generates \( L(G) \setminus \lambda \).
6. Do Problem 6, Section 6.1 of the text.

7. Consider the CFG $G$ whose productions are as below:

\[
\begin{align*}
S & \rightarrow AbB \mid B \\
A & \rightarrow CD \mid a \\
B & \rightarrow S \mid b \\
C & \rightarrow BbS \mid \lambda \\
D & \rightarrow \lambda
\end{align*}
\]

(a) Eliminate $\lambda-$productions from $G$ and write down the resulting grammar.

(b) Eliminate the unit productions from the grammar you have obtained in part (a) and write down the resulting grammar.

(c) Eliminate useless symbols from the grammar you have obtained in part (b) and write down the resulting grammar.

8. Construct a reduced grammar (i.e. a grammar without useless symbols) equivalent to the CFG $G = (\{S, A, B, C\}, \{a, b, c\}, P, S)$, where $P$ is the set of productions

\[
\begin{align*}
S & \rightarrow aA \mid bC \\
A & \rightarrow aSA \mid bAC \\
B & \rightarrow ABc \mid bSC \mid b \\
C & \rightarrow aAC \mid bc
\end{align*}
\]