Homework VI: Due Tuesday, November 21, 4:00 pm. in CS 138 HW box in Room 2108 in HAROLD FRANK HALL.

You can also turn in your homework in class, at the end of Lecture on Tuesday.

Instructions:

• Your solution must be stapled. Put your name and email address on the first page.

• Write your solutions clearly, with appropriate mathematical rigor and care. Justify all steps of your solution. Partially incorrect solutions can still be worth several points, but unjustified answers will result in zero points for the corresponding question.

• You are not allowed to copy or transcribe answers to homework assignments from others or other sources.

• You are not allowed to post solutions of your homework on the Piazza Q&A. Moreover, if you use facts from the online discussion, you should provide your own justification in your solution.

• You are allowed to discuss homework assignments with others, but you must write your answers independently. You should always be able to argue and explain your answers when asked for clarifications.

• Please note that there are no late homeworks allowed.

Homework VI problems:

1. For each word below, determine whether or not it is generated by each CFG given on the right, and draw a derivation tree for it if it is:

   \[ aabb \quad CFG 1. \quad S \rightarrow aSb \mid ab \]
   \[ abaa \quad CFG 2. \quad S \rightarrow aS \mid bS \mid a \]
   \[ abba \quad CFG 3. \quad S \rightarrow aS \mid aSb \mid X ; \quad X \rightarrow aXa \mid a \]
   \[ aaaaa \quad CFG 4. \quad S \rightarrow aAS \mid a ; \quad A \rightarrow SbA \mid SSba \]
2. Consider the CFG $G$ whose productions are as below:

$$
\begin{align*}
S & \rightarrow AbB \mid B \\
A & \rightarrow CD \mid a \\
B & \rightarrow S \mid b \\
C & \rightarrow BbS \mid \lambda \\
D & \rightarrow \lambda
\end{align*}
$$

(a) Eliminate $\lambda$–productions from $G$ and write down the resulting grammar.

(b) Eliminate the unit productions from the grammar you have obtained in part (a) and write down the resulting grammar.

(c) Eliminate useless symbols from the grammar you have obtained in part (b) and write down the resulting grammar.

3. Construct a reduced grammar (i.e. a grammar without useless symbols) equivalent to the CFG $G = (\{S, A, B, C\}, \{a, b, c\}, S, P)$, where $P$ is the set of productions

$$
\begin{align*}
S & \rightarrow aA \mid bC \\
A & \rightarrow aSA \mid bAC \\
B & \rightarrow ABc \mid bSC \mid b \\
C & \rightarrow aAC \mid bc
\end{align*}
$$

4. Construct a grammar in Chomsky Normal Form that generates $L(M)$ where $M$ is the NFA $M = (\{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_4\})$ and $\delta$ is given by

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\lambda$</th>
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<tbody>
<tr>
<td>$q_1$</td>
<td>${q_2}$</td>
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<td>$q_2$</td>
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<td>$q_4$</td>
<td>${q_2}$</td>
<td>${q_4}$</td>
<td>$\phi$</td>
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5. Construct a Chomsky Normal Form grammar for $\{a^mwwRb^m \mid w \in \{a, b\}^*, \ m \geq 1\}$.

6. Consider the grammar $G$ whose productions are

$$
\begin{align*}
S & \rightarrow XZ \mid ZY \\
X & \rightarrow ZX \mid a \\
Y & \rightarrow XZ \mid a \\
Z & \rightarrow YY \mid b
\end{align*}
$$

(a) Use the CYK algorithm (not an algorithm of your own or an adhoc method) to show that $bababa \in L(G)$.

(b) Use part (a) to construct a derivation tree for $bababa$.