1. Suppose in a PDA \( M \), \( \delta(q_1,a,b) = \{(q_1,\alpha),(q_2,\lambda),(q_3,\beta)\} \). Construct the set of all instantaneous descriptions (IDs) \( \mathcal{I} \) such that \( (q_1,aw,bx) \vdash \mathcal{I} \).

2. Give (formally) the transitions rules of a PDA that accepts the language
\[
\{w \in \{a, b\}^+ \mid |w_a| = |w_b| \}
\]

3. Suppose \( w = \text{Yield}(T) \) where \( T \) is the following parse tree in some CFG \( G \):

![Parse Tree](image)

(a) Construct a leftmost derivation for \( w \).

(b) Find 3 different ways of writing \( w \) (i.e. 3 different decompositions or factorizations) in the form \( w = uxvyz \) such that \( xy \neq \lambda \) and \( ux^kyvy^kz \in \mathcal{L}(G) \) for every \( k \geq 0 \).

4. Construct a PDA (in detail) that accepts the language \( \mathcal{L} = \{a^nb^mc^{n+m} \mid n, m \geq 0\} \) over the alphabet \( \{a, b, c\} \).

5. Show that \( \mathcal{L} = \{a^m b^{2m} \mid m \geq 0\} \) is a deterministic CFL.

6. Let \( \mathcal{L}_1 \) be a CFL and \( \mathcal{L}_2 \) be regular. Show that there exists an algorithm to determine whether or not \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \) have a common element.

7. Consider the grammar
\[
\begin{align*}
S & \rightarrow bX \\
X & \rightarrow bXYZ \mid aY \mid b \\
Y & \rightarrow a \\
Z & \rightarrow c
\end{align*}
\]
(a) Give (formally) the transition rules of a PDA $M$ with three states $q_0, q_1, q_f$ that accepts the language generated by this grammar.

(b) Give the sequence of moves $M$ makes in processing the word $w = b^2ac$, using the “$|$” notation.

(c) Construct a derivation for $w$ that corresponds to the above sequence of moves $M$ makes.

(d) Construct the derivation tree corresponding to the derivation in part (c) above.

8. Consider the PDA $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, z\}, \delta, q_0, z, \{q_2\})$ where $\delta$ is given by

\[
\begin{align*}
\delta(q_0, a, a) &= \{(q_0, aa)\} & \delta(q_0, \lambda, a) &= \{(q_1, a)\} \\
\delta(q_0, b, a) &= \{(q_0, ba)\} & \delta(q_0, \lambda, b) &= \{(q_1, b)\} \\
\delta(q_0, a, b) &= \{(q_0, ab)\} & \delta(q_1, a, a) &= \{(q_1, \lambda)\} \\
\delta(q_0, b, b) &= \{(q_0, bb)\} & \delta(q_1, b, b) &= \{(q_1, \lambda)\} \\
\delta(q_0, a, z) &= \{(q_0, az)\} & \delta(q_1, \lambda, z) &= \{(q_2, z)\} \\
\delta(q_0, b, z) &= \{(q_0, bz)\} 
\end{align*}
\]

(a) Starting with

\[
(q_0, abba, z) \vdash (q_0, bba, az) \vdash (q_0, ba, baz) \vdash \cdots
\]

give the remaining sequence of moves the machine makes to accept $abba$.

(b) Is $M$ deterministic? Why?