1. \(L_1\) has a CFG:
   \[
   \begin{align*}
   S &\rightarrow ACA \\
   C &\rightarrow ACA|c \\
   A &\rightarrow a|b \\
   \end{align*}
   \]
\(L_2\) has a CFG:
   \[
   \begin{align*}
   S &\rightarrow ACA \\
   C &\rightarrow c \\
   A &\rightarrow a|b|aA|bA \\
   \end{align*}
   \]
\(L_3\) has a CFG:
   \[
   \begin{align*}
   S &\rightarrow ACA|BCB \\
   C &\rightarrow ACA|BCB|c \\
   A &\rightarrow a \\
   B &\rightarrow a \\
   \end{align*}
   \]
2. Similar to example 8.2 of the book.
3. \(S \rightarrow aB|aS|aAS|aSS\)
   \[
   \begin{align*}
   A &\rightarrow a \\
   B &\rightarrow b \\
   \end{align*}
   \]
4. (a) \(L_1\) is not a CFG. Let \(w = a^mb^{m+1}c^{2m+1} \in L\) and consider the decomposition \(w = uvxyz\). If \(vxy\) lies entirely within the \(a^m, b^{m+1},\) or \(c^{2m+1}\) portions of the string, \(uv^2wx^2y = 2 \not\in L\), since we’re increasing the number of a’s or b’s while leaving the count of c’s unchanged, or increasing the number of c’s while leaving the other two counts unchanged. If \(vxy\) lies both in \(a^m\) and \(b^{m+1}\), \(uv^0xy^0z \not\in L\), since we’ll decrease the number of a’s and/or b’s while leaving the count of c’s unchanged. If \(vxy\) lies both in \(b^{m+1}\) and \(c^{2m+1}\), then v must include at least one b, and so \(uv^0xy^0z \not\in L\), since we’re decreasing the number of b’s and thus the condition that there are more b’s than a’s will not hold.
(b) $L_2$ has a CFG:

\[
S \rightarrow ASC \mid BTC \mid \lambda \\
T \rightarrow BTC \mid \lambda \\
A \rightarrow a \\
B \rightarrow b \\
C \rightarrow c
\]

5. Suppose $A \rightarrow a_1a_2...a_nBb_1b_2...b_n$. Introduce a new variable $A_1$ such that $A_1 \rightarrow a_2...a_nBb_1b_2...b_n$ and $A \rightarrow a_1A_1$. This process continues to remove terminals on the left and on the right.

6. Suppose $m$ is the pumping length. We pick $w$ to be $a^m$. The decomposition is $w = uvxyz$. The length, $|uvxyz| = m^2$ and $|vxy| \leq m$. The pumped string looks like $w = uv^i xy^i z$. If we use $i = 2$ the length of the string will be less than or equal to $m^2 + m$ so we will have:

\[
m^2 < |uv^2 xy^2 z| \leq m^2 + m < m^2 + 2m + 1 < (m + 1)^2
\]

Therefore this word is not in the language.