Homework VIII: Due Tuesday, December 5, 4:00 pm. in CS 138 HW box in Room 2108 in HAROLD FRANK HALL.

You can also turn in your homework in class, at the end of Lecture on Tuesday.

Instructions:

- Your solution must be stapled. Put your name and email address on the first page.

- Write your solutions clearly, with appropriate mathematical rigor and care. Justify all steps of your solution. Partially incorrect solutions can still be worth several points, but unjustified answers will result in zero points for the corresponding question.

- You are not allowed to copy or transcribe answers to homework assignments from others or other sources.

- You are not allowed to post solutions of your homework on the Piazza Q&A. Moreover, if you use facts from the online discussion, you should provide your own justification in your solution.

- You are allowed to discuss homework assignments with others, but you must write your answers independently. You should always be able to argue and explain your answers when asked for clarifications.

- Please note that there are no late homeworks allowed.

Homework VIII problems:

1. Consider the CFG \( G = (\{A_1, A_2, A_3\}, \{a, b\}, P, A_1) \) where \( P \) consists of the productions

\[
\begin{align*}
A_1 & \rightarrow A_1A_3 \mid a \\
A_2 & \rightarrow A_3A_1A_3 \\
A_3 & \rightarrow A_3A_1A_2 \mid b
\end{align*}
\]

(a) Construct a derivation tree \( T \) for \( w = ababab \in L(G) \) over \( G \).

(b) Give a rightmost derivation obtained from \( T \) for \( w \).

(c) Obtain a grammar equivalent to \( G \) which has no left recursive productions.
(d) Put $G$ into Chomsky normal form.
(e) Put $G$ into Greibach normal form.

2. Construct a PDA that accepts the language $L = \{a^nb^mc^n+m \mid n, m \geq 0\}$ over the alphabet $\{a, b, c\}$.

3. Show that $L = \{a^mb^2m \mid m \geq 0\}$ is a deterministic CFL.

4. Let $L_1$ be a CFL and $L_2$ be regular. Show that there exists an algorithm to determine whether or not $L_1$ and $L_2$ have a common element.

5. Consider the grammar

\[
\begin{align*}
S & \rightarrow bX \\
X & \rightarrow bXYZ \mid aY \mid b \\
Y & \rightarrow a \\
Z & \rightarrow c
\end{align*}
\]

(a) Give (formally) the transition rules of a PDA $M$ with three states $q_0, q_1, q_f$ that accepts the language generated by this grammar.
(b) Give the sequence of moves $M$ makes in processing the word $w = b^3ac$, using the “$\mid$” notation.
(c) Construct a derivation for $w$ that corresponds to the above sequence of moves $M$ makes.
(d) Construct the derivation tree corresponding to the derivation in part (c) above.

6. Determine the languages generated by the following two grammars in Greibach Normal Form:

\[
\begin{align*}
S & \rightarrow aSB \mid aB \\
B & \rightarrow b \\
S & \rightarrow aSA \mid bSB \mid a \mid b \\
A & \rightarrow a \\
B & \rightarrow b
\end{align*}
\]

7. Suppose $G$ is a CFG and $w \in L(G)$ with $w \neq \lambda$. How long is a derivation of $w$ in $G$ in terms of $|w|$ if

(a) $G$ is in Chomsky normal form,
(b) $G$ is in Greibach normal form.

Prove your answers.

8. Do Problem 2, Section 7.1 of the text.