Adams Methods

Interpolating polynomial through f(k past values)
Integrate take interpolating polynomial.

AB-1

$$\phi_{n-1}(t) = f(t_{n-1})$$

$$y(t_n) = y(t_{n-1}) + \int_{t_{n-1}}^{t_n} f(t_{n-1})\,dt$$

$$\Rightarrow y_n = y_{n-1} + \int_{t_{n-1}}^{t_n} \phi_{n-1}(t)\,dt = y_{n-1} + h_n f_{n-1}$$

AB-2

$$\phi_{n-1}(t) = f(t_{n-1}) + f[t_{n-1}, t_{n-2}](t - t_{n-1})$$

$$= f(t_{n-1}) + \left(\frac{f(t_{n-1}) - f(t_{n-2})}{h_{n-1}}\right)(t - t_{n-1})$$

$$y_n = y_{n-1} + \int_{t_{n-1}}^{t_n} \phi_n(t)$$

$$= y_{n-1} + h_n f_{n-1} + \left(\frac{f_{n-1} - f_{n-2}}{h_{n-1}}\right)\frac{h_n^2}{2}$$

~~BDF Methods~~

AM-1

$$\phi_n = f_n, \quad y_n = y_{n-1} + h_n f_n$$

AM-2

$$\phi_n = f_n + \left(\frac{f(t_n) - f(t_{n-1})}{h_n}\right)(t - t_n)$$

$$\Rightarrow y_n = y_{n-1} + h_n f_n \cancel{+} - \left(\frac{f(t_n) - f(t_{n-1})}{h_n}\right)\left(\frac{h_n^2}{2}\right)$$

BDF

Interpolate through the last k y's.

Differentiate the interpolating polynomial.

Implicit Euler

$$\phi_n = y_n + \left(\frac{y_n - y_{n-1}}{h_n}\right)(t - t_n)$$

$$\phi' = \left(\frac{y_n - y_{n-1}}{h_n}\right) = f_n$$

2nd order BDF

$$\phi_n = y_n + \left(\frac{y_n - y_{n-1}}{h_n}\right)(t - t_n) + \left(\frac{\frac{y_n - y_{n-1}}{h_n} - \frac{y_{n-1} - y_{n-2}}{h_{n-1}}}{h_n + h_{n-1}}\right)(t - t_n)(t - t_{n-1})$$

$$\phi' = \left(\frac{y_n - y_{n-1}}{h_n}\right) + \left(\frac{\frac{y_n - y_{n-1}}{h_n} - \frac{y_{n-1} - y_{n-2}}{h_{n-1}}}{h_n + h_{n-1}}\right)\left[(t - t_n) + (t - t_{n-1})\right]$$

$$\phi'(t_n) = f_n \Rightarrow \left(\frac{y_n - y_{n-1}}{h_n}\right) + \left(\frac{\frac{y_n - y_{n-1}}{h_n} - \frac{y_{n-1} - y_{n-2}}{h_{n-1}}}{h_n + h_{n-1}}\right) h_n = f_n$$

Constant h check!

$$\frac{y_n - y_{n-1}}{h} + \frac{1}{2h}\left(y_n - y_{n-1} - (y_{n-1} - y_{n-2})\right) = f_n$$

$$\Rightarrow \frac{3}{2}y_n - 2y_{n-1} + \frac{1}{2}y_{n-2} = hf_n$$

$$\Rightarrow y_n - \frac{4}{3}y_{n-1} + \frac{1}{3}y_{n-2} = \frac{2}{3}hf_n \checkmark$$