Homework 2, due Wednesday January 25

1. (1 point each part) For each of the following constant-coefficient systems $y' = Ay$, determine if the system is stable, asymptotically stable, or unstable:

   (a)
   $$A = \begin{pmatrix} -5 & 0 \\ 0 & -0.5 \end{pmatrix}$$

   (b)
   $$A = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$$

   (c)
   $$A = \begin{pmatrix} -1 & 10 \\ 0 & -2 \end{pmatrix}$$

   (d)
   $$A = \begin{pmatrix} 0 & -1 \\ 5 & 0 \end{pmatrix}$$

2. (5 points) Consider the method of lines applied to the diffusion equation in one space dimension, $u_t = au_{xx}$, with $a > 0$ a constant, $u = 0$ at $x = 0$, $x = 1$ for $t \geq 0$, and with given initial values. Formulate the method of lines using the central difference approximation to the derivative u_{xx}, as in Example 1.3 of the text (as we did in Lecture 1), to arrive at a linear constant coefficient ODE system $y' = Ay$ with A symmetric. Find the eigenvalues of A to determine whether the problem is unstable, stable or asymptotically stable. (Hint: Try eigenvector $\mathbf{v}^k$ in the form $v_i^k = \sin(ik\pi\Delta x)$, $i = 1, \ldots, m$, for $1 \leq k \leq m$, and use the sum of angles formula for sin).

3. (5 points) Consider the method of lines applied to the advection equation in one space dimension, $u_t + au_x = 0$, with $a > 0$ a constant, on the spatial domain $0 \leq x \leq 1$, with boundary condition $u = 0$ at

$x = 0$, and with given initial values. Formulate the method of lines using the first order backward difference approximation to the derivative u_x, , to arrive at a linear constant coefficient ODE system $y' = Ay$. Find the eigenvalues of A to determine whether the problem is unstable, stable or asymptotically stable. Next, consider approximating the derivative u_x by the first order forward difference approximation (see the Scholarpedia MOL article for the definition of the forward difference approximation). For this, use the boundary condition $u = 0$ at $x = 1$. Find the eigenvalues of A to determine whether the problem is unstable, stable or asymptotically stable.

4. (6 points) Write a well-structured MATLAB code implementing the explicit Euler method for systems of standard-form ODEs. Apply the code to the following problem. (Note that the system is uncoupled, but you are to treat it in your code as if it were coupled.)

$$\begin{aligned} y_1' &= -y_1 \\ y_2' &= -100(y_2 - t^2) + 2t \end{aligned}$$

for $0 \le t \le 1$, with initial value $y_1 = 1$, $y_2 = 2$. Try this for 2 sequences of stepsizes:

- $h = .01$
- $h = .05$

Plot the solutions obtained. Explain your results.