MEE 210B

Homework # 6

Due March 12, 2012, 9am in class or 8:30am in the homework box

Note: Please turn in the theory problems separately from the programming problems.

1. Consider the Runge-Kutta method defined by

$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & \frac{1}{6} & \frac{2}{3} & 0 & \frac{1}{6} \end{array}$$

   (a) (1 point) Write the formula which corresponds to this Runge-Kutta matrix

   (b) (2 points) Using the order conditions from the book, find the order of this method.

2. (3 points) Write the following in Runge-Kutta matrix form, and find its order using the order conditions from the book:

$$\begin{aligned} y_{n+\frac{2}{3}} &= y_n + \frac{h}{3}[f(y_{n+\frac{2}{3}}) + f(y_n)] \\ y_{n+1} &= y_n + \frac{h}{4}[3f(y_{n+\frac{2}{3}}) + f(y_n)] \end{aligned}$$

3. (3 points) It has been argued that displaying absolute stability regions as in Figure 4.4 of the text is misleading: since a step of an s-stage explicit method costs essentially s times a forward Euler step, its stability region should be compared with what forward Euler can do in s steps. Thus, the *scaled stability region* of an s-stage explicit method is the stability region shrunk by a factor s. For all Runge-Kutta methods with $p = s$, $s = 1, 2, 3, 4$, plot the scaled stability regions. Compare to the forward Euler stability region.

4. Consider the 2-stage second order implicit Runge-Kutta method defined by the matrix

$$\begin{array}{cc|cc} \gamma & \gamma & 0 \\ 1 & 1-\gamma & \gamma \\ \hline & 1-\gamma & \gamma \end{array}$$

where $\gamma = \frac{2-\sqrt{(2)}}{2}$.

(a) (5 points) Write down the method corresponding to these coefficients, and the Newton iteration for this method. Explain how to make use of the structure of this method to solve the linear system in the Newton iteration as efficiently as possible.

(b) (2 points) Does the method have stiff decay? Why or why not? Explain why, and when, stiff decay is a desirable property.