Random Waypoint Considered Harmful

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***Abstract*— This study examines the random waypoint model widely used in simulating mobile ad hoc networking research. Our findings show that this model fails to provide a "steady state" in that the average nodal speed consistently decreases over time, and therefore should not be directly used for simulation. We show how unreliable and misleading results can be obtained by using this model. In particular certain ad hoc routing metrics can drop by as much as 40% over the course of a 900-second simulation using the random waypoint model. We give both an intuitive and a formal explanation as to the reason behind this phenomenon. We also propose a simple fix of the problem and discuss a few alternatives. Our modified random waypoint model is able to reach a steady state and simulation results are presented.**
Methods Keywords: simulation, statistics.

***Index Terms*—random waypoint model, mobility, ad hoc routing.**

I. INTRODUCTION

Mobile systems are characterized by the *movement* of their constituents. The nature of movement—its speed, direction, and rate of change—can have a dramatic effect on protocols and systems designed to support mobility. Unfortunately, movement in the physical world is often unrepeatable. Live use of a mobile system can provide meaningful insight, but cannot form the sole basis of experimental evaluation.

Instead, the mobile computing community has turned to *simulating* the movement of nodes and users. Of course, one must derive a model of movement to drive such a simulation. By far the most common of these is the *random waypoint* model. This model was first used by Johnson and Maltz in the evaluation of Dynamic Source Routing (DSR) [1], and was later refined by the same research group [2]. The refined version has become the *de facto* standard in mobile computing research. For example, ten papers in ACM MobiHoc 2002 considered node mobility, and nine of them used the random waypoint model.

In this model, nodes in a large "room" choose some destination, and move there at a random speed chosen from $(0, V_{max}]$, where V_{max} is the *maximum speed* of the simulation. Often, the model will be described as having an average speed of $\frac{V_{max}}{2}$. This model is expected to maintain this average speed as simulation progresses, and simulation results are almost always in the form of an average over a period of time. Such averages only make sense if the simulation reaches a steady state. Unfortunately, this is not the case. The fact is that as simulated time progresses, the collection of nodes moves more slowly; more and more nodes become "stuck" travelling long distances at low speeds. Thus the model fails to provide a steady state in terms of average speed. The overheads and performance of mobile systems usually depend strongly on node mobility. In light of this, random waypoint can generate misleading or incorrect results. In particular, time-average results change drastically over time; the longer we run the simulation, the further results deviate.

This paper presents an analysis of a generalized random waypoint model that predicts the average speed of nodes in the simulation. This analysis closely matches the actual model. There are many ways to correct the model. One simple way is to limit the minimum speed, as well as the maximum. The paper compares this simple improvement to the original model, and demonstrates its marked improvement in stability over the course of the simulation. We also explore the impact of instability on two ad hoc routing protocols, DSR [1], [2] and AODV [3], [4]. Either protocol can produce better packet delivery rates and delays, depending on the average speed of the nodes during simulation.

The results presented in this paper highlight our belief that simulation studies should be done with great caution. Since certain assumptions associated with a simulation model is "hidden", it requires careful examination to ensure that a simulation model is actually doing what we believe it is doing. This work does not propose a mobility model to more accurately reproduce real movement. Rather, we present a simple modification to random waypoint to produce more stable movement patterns suited for simulation studies.

The rest of the paper is organized as follows. In Section II we describe in detail the observed problem with the random waypoint model via an intuitive explanation and a formal analysis. Section III provides a simple improvement to the original model and presents simulation results. Discussion on alternative solutions and related works are given in Section IV. Section V concludes the paper.

II. An In-Depth look into the random waypoint model

A. *The problem and an intuitive explanation*

The performance measures of ad hoc routing protocols are directly affected by the underlying mobility model used. One of the most important parameters of a mobility model is the node speed, either in the form of a constant value or in the form of a set of values that determine a certain distribution [2], [4], [5], [6], [7]. Users should be able to adjust this parameter in order to compare the performance of routing protocols under different nodal speed. In doing so, a mobility model is naturally expected to reach a certain *equilibrium* where the instantaneous average node speed is stablized around a constant. In the random waypoint model, this average is often believed to be half of the maximum speed — simply because node speeds are chosen from a uniform distribution $(0, V_{max}]$ — or some value between 0 and V_{max} if we consider positive pause time. Moreover, this average is believed to be reached at the onset of the simulation. Based on such expectations, simulation studies often include comparisons between varying values of V_{max}. For each value of V_{max}, simulation results (e.g., routing overhead, packet delivery ratio, etc.) are often in the form of averages over a period of time (e.g., 900 seconds used by studies in [2], [4]).

But are these expectations really justified? Let's formally define the *instantaneous average nodal speed* of mobility scenarios generated by the random waypoint model in the following way:

$$\bar{v}(t) = \frac{\sum_{i=1}^{N} v_i(t)}{N}$$

where N is the total (fixed) number of nodes in the scenario, and $v_i(t)$ is the speed of the i^{th} node at time t. Using this definition, an example generated by the random waypoint model with maximum speed of 20(m/s) and zero pause time is plotted in Fig 1. This is the average over 30 different scenarios and the average speed is calculated every second.

We see that the instantaneous average speed is consistently decreasing. As we will show in the next subsection, this average under the random waypoint model would eventually approach zero! We can easily imagine how erroneous and misleading results can be obtained if we use this model to evaluate the performance of ad hoc routing protocols. We will show specific instances in Section III.

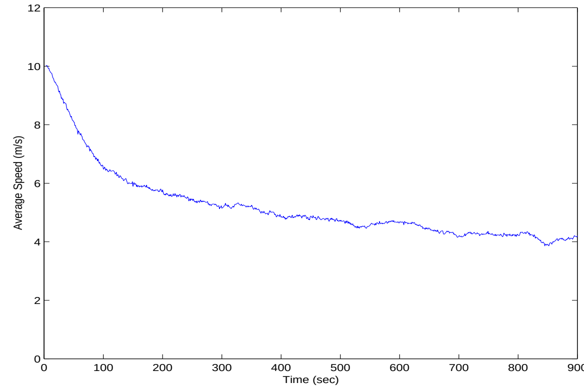


Fig. 1. Average speed decay (speed=$(0, 20]$, pause=0)

An intuitive explanation for this is as follows. The random waypoint model chooses a destination and a speed for a node at random, and the node will keep moving at that speed until it reaches the destination. Given such, a node with a slow speed and a far-away destination may take a long time to finish the trip or may never reach the destination within simulation time.[1] If they do reach the destination they will be assigned another possibly higher random speed, but nodes like this can be "trapped" to these slow journeys for significant amount of time and therefore dominate the average nodal speed. As the simulation time goes on, on average more and more nodes will be trapped to slower trips, thus causing the speed decay observed in Fig 1. Note that running the simulation longer only cause the average to reduce even more. Before we move on to present our fix for this problem, it helps to take a closer look at the probabilistic reasons behind this problem in a more formal way, which we discuss in the next subsection.

B. *A formal analysis of the problem*

Here we reiterate the procedure of the random waypoint model. We first choose a rectangular area of size $X_{max} \times Y_{max}$, and the total number of nodes N in the area. We then choose a random initial location (x, y) for each node, where x and y are both uniformly distributed over $[0, X_{max}]$ and $[0, Y_{max}]$, respectively. Every node is then assigned a destination (x', y'), also uniformly distributed over the two-dimensional area, and a speed v, which is

[1] For example, using an area of $1500m \times 500m$, and speed range of $(0, 20]$(m/s), if a destination is chosen $1000m$ away and the speed is chosen to be 0.1m/s, then the travel time would be 10000 seconds.

uniformly distributed over $(0, V_{max}]$.[2] V_{max} is the user-assigned maximum allowed speed. A node will then start travelling toward the destination on a straight line at the chosen speed v. Upon reaching the destination (x', y'), the node stays there for a specific pause time.[3] Upon expiration of the pause time, the next destination and speed are again chosen in the same way and the process repeats until the simulation ends.

For simplicity and clarity of our illustration, we will analyze a modified or generalized version of the above model with the following assumptions. We note that our conclusion remains true for the original random waypoint model, but the simplifying assumptions help to isolate and emphasize the key reasons behind the diminishing average nodal speed.

(A1) Instead of confining the nodal movement to a rectangular area of $X_{max} \times Y_{max}$, we will assume that nodes move in an unlimited, arbitrarily large area. Given the current location of a node, the destination is chosen uniformly from a circle of radius R_{max} centered at the current node location. The rationale behind this assumption is that it allows us to easily derive the distribution of travel distances. In the original model the travel distance is dependent on the node location due to the limited movement area. We emphasize again that this assumption only helps simplify the analysis but does not change the ultimate conclusion.[4]

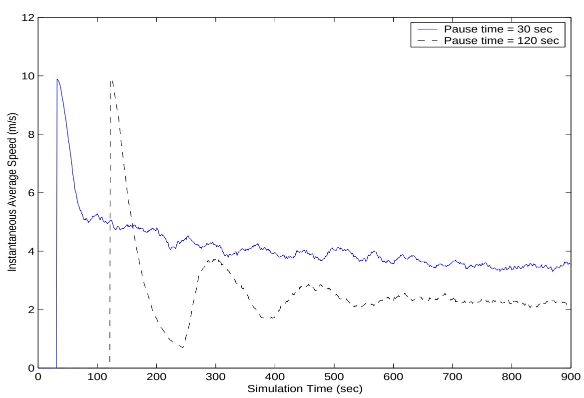


Fig. 2. Average speed decay with various pause time (speed=$(0, 20]$)

(A2) We assume that all pause times are zero. Again this assumption is made because the pause time is not key to the speed decay, and thus eliminating it simplifies the analysis. Fig.2 shows how the average node speed decays with non-zero pause times. Even though longer pause times lead to fluctuations in the beginning, such effect is gradually reduced and the average node speed (or the envelope of the fluctuation) still quickly decays as time progresses.

(A3) We assume that for each travel the node speed is chosen uniformly from the interval $[V_{min}, V_{max}]$ instead of $(0, V_{max}]$, where $V_{min} > 0$. This is because the latter is a limiting case of the former, and thus can be easily derived from results obtained for the former. By this assumption we also imply that the user can specify both the *minimum* and *maximum* allowed speed. Given this assumption the probability density function (pdf) of the nodal speed V is

$$f_V(v) = \frac{1}{V_{max} - V_{min}}, \quad V_{min} \le v \le V_{max}.$$

Note that the choice of speed and the choice of destination (therefore travel distance) are mutually independent.

Then after some derivation we can obtain the pdfs and expectations for the travel distance and the travel time summarized as follows (see the Appendix for details).

1) The pdf of the travel distance D is

$$f_D(d) = \frac{2d}{R_{max}^2}, \quad 0 \le d \le R_{max}$$

and the expected travel distance is $E[D] = \frac{2}{3} R_{max}$.

2) The pdf of the travel time T (note that as a random variable, $T = \frac{D}{V}$) is

$$f_T(t) = \begin{cases} \frac{2(V_{max}^2 + V_{min}^2 + V_{max}V_{min})}{3R_{max}^2} \cdot t & 0 \le t \le \frac{R_{max}}{V_{max}} \\ \frac{2R_{max}}{3(V_{max}-V_{min})} \cdot \frac{1}{t^2} - \frac{2V_{min}^3}{3R_{max}^2(V_{max}-V_{min})} \cdot t & \frac{R_{max}}{V_{max}} \le t \le \frac{R_{max}}{V_{min}} \\ 0 & t \ge \frac{R_{max}}{V_{min}} \end{cases}$$

and the expected travel time is

$$E[T] = \frac{2R_{max}}{3(V_{max} - V_{min})} \ln\left(\frac{V_{max}}{V_{min}}\right). \qquad (1)$$

This pdf is shown in Fig. 3.

3) Given the above results, we can compute the time average of the speed for a given node, $\bar{V}$, as follows, assuming the instantaneous node speed of $v(s)$ at

[2] Note that it is also often presented as a uniform distribution over $[0, V_{max}]$. However in actual simulation, e.g., the `setdest` mobility generation utility in ns-2 [8], zero is always eliminated to avoid division by zero.

[3] In some literatures, random amount of pause time with uniform or exponential distribution is used.

[4] The boundary effect of the current random waypoint model has be studied in [6], [7], [9]. However, with or without boundary does not fundamentally affect our analysis here.

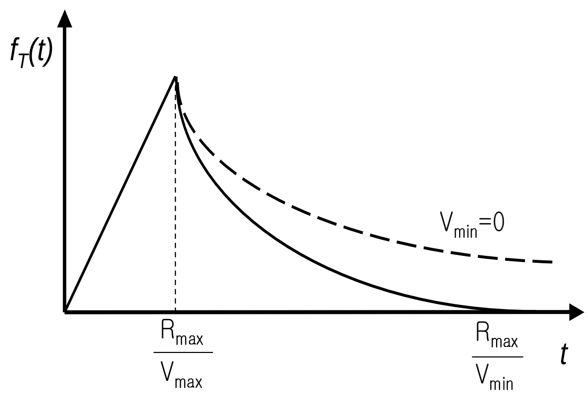


Fig. 3. The pdf of travel time T

time s (note that since each node moves independently, it suffices to consider a single node):

$$\begin{aligned}\bar{V} &= \lim_{S\longrightarrow\infty}\frac{1}{S}\int_0 Sv(s)ds \\ &= \lim_{S\longrightarrow\infty}\frac{\sum_{k=1}K(S)d_k}{\sum_{k=1}K(S)t_k} \\ &= \lim_{S\longrightarrow\infty}\frac{\frac{1}{K(S)}\cdot\sum_{k=1}K(S)d_k}{\frac{1}{K(S)}\sum_{k=1}K(S)t_k} \\ &= \frac{E[D]}{E[T]}=\frac{V_{max}-V_{min}}{\ln\left(\frac{V_{max}}{V_{min}}\right)}. \end{aligned} \quad (2)$$

Here $K(S)$ is the total number of trips taken within time S, including the last one that may be incomplete. d_k and t_k are the travel distance and the travel time of the k^{th} trip, respectively. The first equality in (2) follows from the assumption that ensemble averages equal time averages.

From the distribution $f_T(t)$, we see that as V_{min} approaches 0, the tail distribution of $f_T(t)$ decays approximately according to $\frac{k}{t^2}$ as $t\longrightarrow\infty$. Indeed if $V_{min}=0$ then T has a heavy tailed distribution [10] since $f_T(t)$ becomes $\frac{2R_{max}}{3V_{max}}\frac{1}{t^2}$. Similarly, from Equation (1), $E[T]\longrightarrow\infty$ as $V_{min}\longrightarrow 0$, and from (2) $\bar{V}\longrightarrow 0$ as $V_{min}\longrightarrow 0$. In other words, this means that as the minimum speed approaches zero, the travel time has a higher and higher probability density of being very large, with an expected travel time approaching infinity. At the same time, the expected node speed approaches zero over time.[5]

[5]One can easily check using standard methods that if we take two uniformly distributed nonnegative continuous random variables (X_1 and X_2, respectively) and take a third random variable (X_3) to be the ratio of the first two ($X_3 = X_1/X_2$), then if the denominator random variable (X_2) is defined over an interval containing the origin, X_3 would indeed be heavy-tail distributed.

Comparing the average speed $\bar{V}$ to the initial average speed $\bar{V}_{init}$ defined as $\frac{V_{max}+V_{min}}{2}$ and setting $x=\frac{V_{max}}{V_{min}}>1$, we have

$$\begin{aligned}\frac{\bar{V}}{\bar{V}_{init}} &= \frac{\frac{V_{max}-V_{min}}{\ln\frac{V_{max}}{V_{min}}}}{\frac{V_{max}+V_{min}}{2}} \\ &= \frac{2(\frac{V_{max}}{V_{min}}-1)}{(\frac{V_{max}}{V_{min}}+1)\ln(\frac{V_{max}}{V_{min}})} \\ &= \frac{2(x-1)}{(x+1)\ln x}=g(x)\end{aligned}$$

Note that $g^{'}(x)<0$ ($x>1$), $\lim_{x\longrightarrow 1}g(x)=1$ and $\lim_{x\longrightarrow\infty}g(x)=0$. Therefore

$$\bar{V}\leq\bar{V}_{init}\quad \text{(equality holds when } V_{max}=V_{min}).$$

This means that the average nodal speed over time is always less than the initial average speed, unless the speed is constant. This also means that there will always be a period of speed decay at the beginning of the simulation until the average speed settles around $\bar{V}$. The distinction is that if V_{min} takes a positive value, then the average speed eventually stablizes to a positive $\bar{V}$, whereas if $V_{min}=0$ then the average speed will continue to decrease as time goes on. Even when V_{min} is positive, it may take a long time before the average node speed stablizes if V_{min} is very small. In general, the smaller V_{min}, the longer the decay period.

The analysis and discussions in this section suggest two things:

1) In order to reach a positive average speed, one solution is to specify a positive minimum speed for the mobility model, and this minimum speed cannot be too close to zero because it is desirable for any simulation model to reach stability as soon as possible. Subsequently, for simulation comparison purposes, the average nodal speed has to be carefully calculated rather than taking the simple average of maximum and minimum speed values. Table I shows a list of various speed range and the corresponding speed averages. $\bar{V}_{init}$ is the initial average speed, taken as the median of minimum and maximum speed. $\bar{V}$(Eq.2) is the steady-state average speed calculated by Eq. (2) following assumptions (A1)-(A3). $\bar{V}$(simulation) is the time average speed of 50 nodes for 1000 seconds after the first 500 second warm-up. Simulation results are averaged over 10 different scenarios generated by the random waypoint model in 1500m $\times$ 500m area with the modification of positive minimum speed. σ is the standard deviation. As shown in Table I, all

simulated average speeds except in the case of V_{min} = 0 are very close to our analysis despite of our assumptions $(A1)$-$(A3)$. Therefore this estimate can be used in simulation studies to quantify the average nodal speed. In the case of V_{min} = 0, the average speed decayed from $\bar{V}_{init}$ but did not converge yet to the expected average speed which is zero. Simulation for longer time shows that the average speed of the speed range $(0, 20]$ keeps decaying. For example, over 10 scenarios, the average speed was 4.07(m/s) (σ = 0.48) at t = 1000 seconds, 3.16(m/s) (σ = 0.61) at t = 5000 seconds, and 2.85(m/s) (σ = 0.51) at t = 10000 seconds. It decays slowly but consistently.

2) It is a common practice in the traditional simulation community to "warm-up" a simulation (also known as *initial data deletion*) aimed at eliminating the effect of the transient part by discarding the data from the initial period of a certain length. This is usually done to ensure that the system being simulated has entered steady state. As shown in Table I, the average node speed quickly decays from the initial average speed $\bar{V}_{init}$ and converges to the expected steady-state average speed $\bar{V}$(sim) with reasonably small standard deviations after some amount of warm-up. The discussion here provides another reason why a simulation using such mobility models needs to be warmed-up so that the average nodal speed converges. The actual amount to be discarded in this particular case depends on the minimum speed.

TABLE I

$\bar{V}_{init}$, $\bar{V}$(BY EQ.(2) AND SIMULATION) OF VARIOUS SPEED RANGE

$(unit : m/s)$

Speed range	$\overline{V}_{init}$	$\overline{V}$ (Eq.2)	$\overline{V}$ (sim)
(0,20]	10	0	4.23 (σ=0.27)
[1,19]	10	6.11	6.17 (0.19)
[2,18]	10	7.28	7.33 (0.19)
[3,17]	10	8.07	8.00 (0.21)
[4,16]	10	8.66	8.70 (0.14)
[5,15]	10	9.10	9.09 (0.12)
[6,14]	10	9.44	9.45 (0.09)
[7,13]	10	9.69	9.69 (0.08)
[8,12]	10	9.87	9.87 (0.05)
[9,11]	10	9.97	9.97 (0.02)
Speed range	$\overline{V}_{init}$	$\overline{V}$ (Eq.2)	$\overline{V}$ (sim)
[1,21]	11	6.57	6.49 (σ=0.22)
[2,22]	12	8.34	8.42 (0.28)
[3,23]	13	9.82	9.80 (0.25)
[4,24]	14	11.16	11.24 (0.22)
[5,25]	15	12.43	12.50 (0.27)
[6,26]	16	13.64	13.70 (0.25)
[7,27]	17	14.82	14.83 (0.22)
[8,28]	18	15.96	16.03 (0.29)
[9,29]	19	17.09	17.23 (0.25)
[10,30]	20	18.20	18.27 (0.27)

III. IMPROVED MODELS AND SIMULATION

In this section we propose a simple method to modify the random waypoint model so that the mobility scenario reaches a steady state in terms of node speed after a quick warm-up period. We further illustrate the significance of this improvement by comparing simulation results generated by this modified model to that generated by the original random waypoint model. We present results for both DSR and AODV over a range of commonly accepted performance metrics. We show that the modified model provides more reliable time-average measures, and that the original model can generate misleading results.

A. Improvement

There are many solutions to the average speed decay problem. Here we suggest one of the simplest improvements to mitigate the decay of average node speed and leave the discussion on alternative methods to the next section.

Our analysis in section II suggests that one solution is to set a non-zero minimum speed. As shown in Table I, the average speed increases proportional to the logarithmic scale which is implied by Eq.(2). General case of speed range $(0, 20]$ results in zero average speed whereas speed range $[1, 19]$ leads to 6.11(m/s). Our study shows that this improvement makes the simulation results quickly converge to the constant and stable level after a certain length of warm-up period. Let us define "settling time" as the time to approach within 10% of the steady-state average speed. Then the settling time of scenarios $[0, 20]$ is almost *impossible* to measure due to the continuous speed decay from 10(m/s) to zero. Note that the initial median speed of both speed ranges is 10. On the contrary, our scenarios of $[1, 19]$ settle down after only 142 seconds, which can be considered as within a warm-up period during 900-second simulation. This can guarantee the fairness of performance comparison regardless of the simulation time elapsed. Next we show simulations based on this solution.

B. Simulation Environment

To maintain consistency with other research results, we employed the *ns-2* simulator [8], using the same node movement (each scenario 900 seconds long) and traffic data as in the previous work on performance comparison by Broch et al. [2] as well as our own. In the case of positive minimum speed such as a speed range $[1, 19]$, we generated mobility data using a simple modified version of `setdest` [8]. In these scenarios, 50 nodes move in a 1500m $\times$ 300m rectangular area at uniformly distributed speeds. Since *pause time* does not have a significant effect on our analysis, we set the pause time to zero in all scenarios. For each set of parameters, we ran 30 different scenarios.[6] The scenarios in [2] we used have a speed range $(0, 20]$ for 900-second simulation runtime. Our scenarios consist of various range of speed including $(0, 20]$. As traffic data, we also chose the same constant bit rate (CBR) scenario as in [2]. Unlike the previous comparisons by Broch et al. [2] and by Perkins et al. [4] with various number of sources and bit rates, we only used CBR scenarios of 30 sources and each source node transmits data of 64 bytes per packet at a rate of 4 packets per second. It is also very important to note that the presentations of our results are all in terms of *time* rather than *pause time* which all previous works used as a changing factor. In other words, *one curve* of our results corresponds to *one point* of the previous works [2], [4].

C. Metrics

To compare our work to the results in the previous works of performance comparison [2], [4], we adopted the following metrics. As mentioned earlier, we calculate these metrics every 100 seconds – e.g., we get 9 data points for a simulation of 900 seconds – so that we can observe changes in the measures in addition to the single average over the entire simulation duration.

- *Average node speed*: The average speed of all nodes is calculated every 100 seconds using Eq.(1). The average node speed is a major factor in our analysis, since how it is changing with respect to *time* affects other metrics.
- *The number of routing overhead packets*: This includes all packets generated by a routing protocol to discover or maintain routes. Each hop taken by a packet is counted separately. So a packet that traverses four hops counts as four overhead packets. All our simulation results in scenarios with speed range $(0, 20]$ and 900 seconds runtime agree (as a single total time average) with that by Broch et al. in [2].
- *Routing overhead packets in bytes*: This metric is the same as the previous one except for the different unit of metric, which is in bytes rather than the number of packets. For example, a 100-byte packet that traverses four hops counts as 400 bytes.
- *The number of dropped data packets*: Due to an error in the physical layer or upper layers, some of transmitted data packets cannot be delivered to a destination node. In this case, a router between a source and a destination discards the packet, and it is counted as one dropped data packet. In our simulation, when averaged every 100-second interval, the number of the transmitted data packets during the interval is in general equal to or a little less than the sum of the received data packets and the dropped packets, since some of packets are still in the middle of network.
- *Data packet delay*: Packet delay is the time elapsed for a data packet to be transmitted from a source node to a destination node. It is calculated only when a packet is successfully delivered to a destination node. This metric is also counted every 100 seconds and averaged over all packets counted.

[6] There are only 10 scenarios of a speed range $(0, 20]$ and pause time 0 in [2]. To obtain more general results, we generated 20 more scenarios and averaged all 30 scenarios.

D. Simulation Results

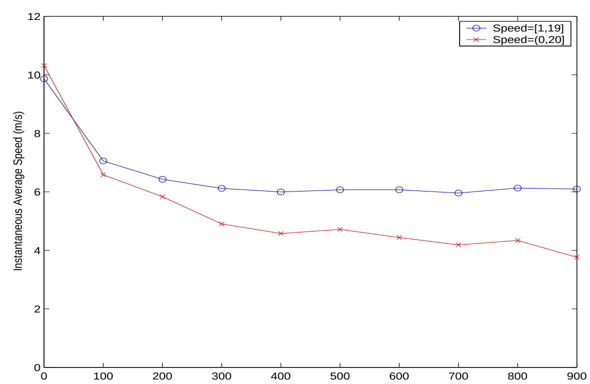


Fig. 4. Average Node Speed in 900-second Simulation

Fig.4 demonstrates how the average node speed changes during the 900-second simulation as time goes on. As shown in Table I in section II, the average node speed in $[1, 19]$ scenarios rapidly converges to the expected average speed 6.11(m/s) and stablizes at that level, whereas the average node speed in $(0, 20]$ scenarios continuously decreases. Since node speed directly affects the performance comparison of ad hoc routing protocols, the *decaying* average node speed during a simulation will result in varying performance measures over time within the same simulation. If we only look at the time average over total simulation runtime, simulation duration becomes a dominating factors affecting the comparison results.

Our simulation results for DSR and AODV from the 900-second scenarios (as in [2], [4]) with speed range $(0, 20]$ and zero pause time, along with results from 900-second scenarios with speed range $[1, 19]$ are shown in Fig.5. We make the following observations.

Firstly, in all four metrics, the performance measures resulted from a speed range $[1, 19]$ were stablized after an initial warm-up period. On the other hand, all the performance measures resulted from speed range $(0, 20]$ continuously decrease as the simulation time progresses, particularly obvious in routing overhead (Fig.5(a) and (b)) and number of dropped packets (Fig.5(c)).

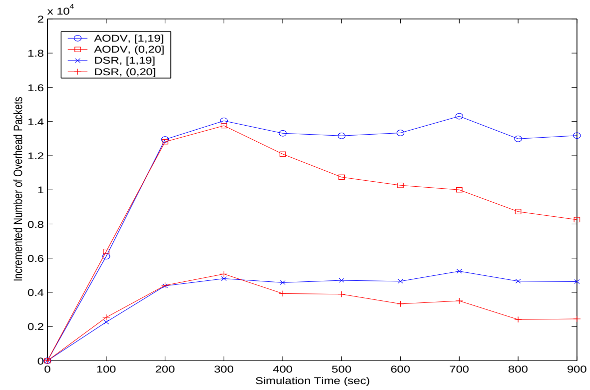
Secondly, in Fig.5(c) after 300 seconds, the difference between DSR and AODV stays relatively constant under speed range $[1, 19]$, while speed range $(0, 20]$ produces two curves that gradually move closer together and even cross each other at some point due to the reduced speed. On the contrary to Fig.5(c), the performance of DSR and AODV are nearly indistinguishable under *both* speed range $[1, 19]$ and $(0, 20]$ in Fig.5(d).

These observations raise questions on the performance comparison of routing protocols since (1) the performance of routing protocols vary depending on the speed range and (2) the speed range (or the instantaneous speed) drastically and consistently decays even during the same simulation. This implies that the performance measures are more closely related to the *instantaneous* average node speed rather than the *maximum* node speed here losing its significance for the performance comparison. Thus, it motivates us to further reveal the relationship between the observed metrics and the instantaneous average node speed.

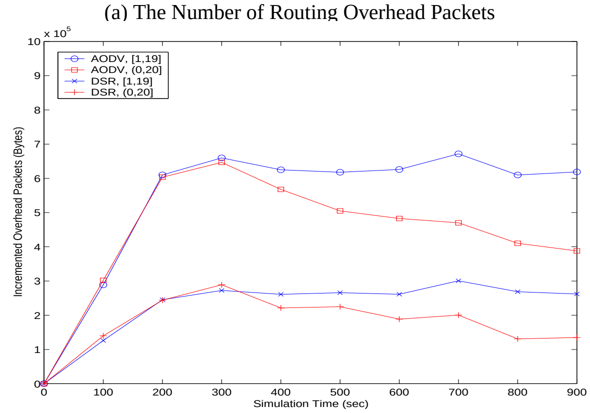
Fig.6 shows how the metrics of DSR and AODV change as the instantaneous average node speed varies. To obtain various instantaneous average node speeds[7], different maximum speeds (1, 5, 10, 15, 20, 25 and 30(m/s))[8] were applied with a minimum speed zero. Each maximum speed were used to generate 10 scenarios, so total 70 different scenarios of simulations were executed. Each point in the Fig.6 corresponds to the average over 10 different

[7]The metric values of an ad hoc routing protocol are directly related to the *instantaneous* node speed rather than the *maximum* speed. Consequently the metric change in terms of the instantaneous average node speed should be a *unique* property of a specific ad hoc routing protocol, and thus can be used to fairly compare the performance of different routing protocols.

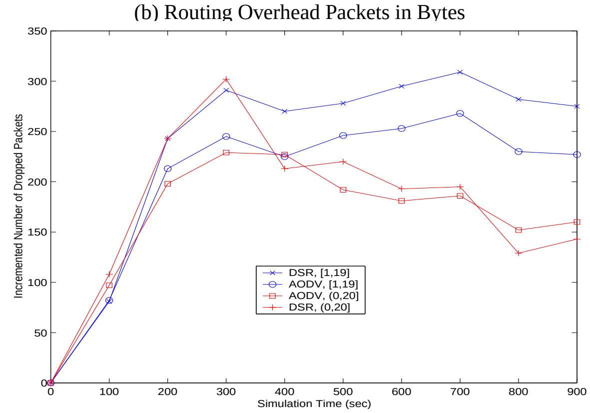
[8]Note that maximum speeds of 25 or 30(m/s) were seldom considered in simulation studies probably because they are considered too high for mobile networking systems (roughly correspond to 57 and 66 mph). However, they are used here to generate a wider range of instantaneous average node speed.



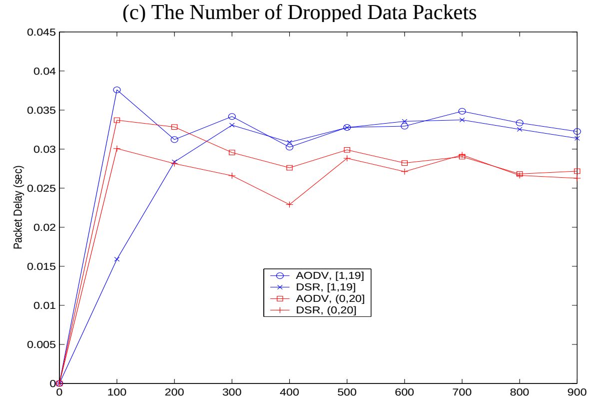
(a) The Number of Routing Overhead Packets



(b) Routing Overhead Packets in Bytes

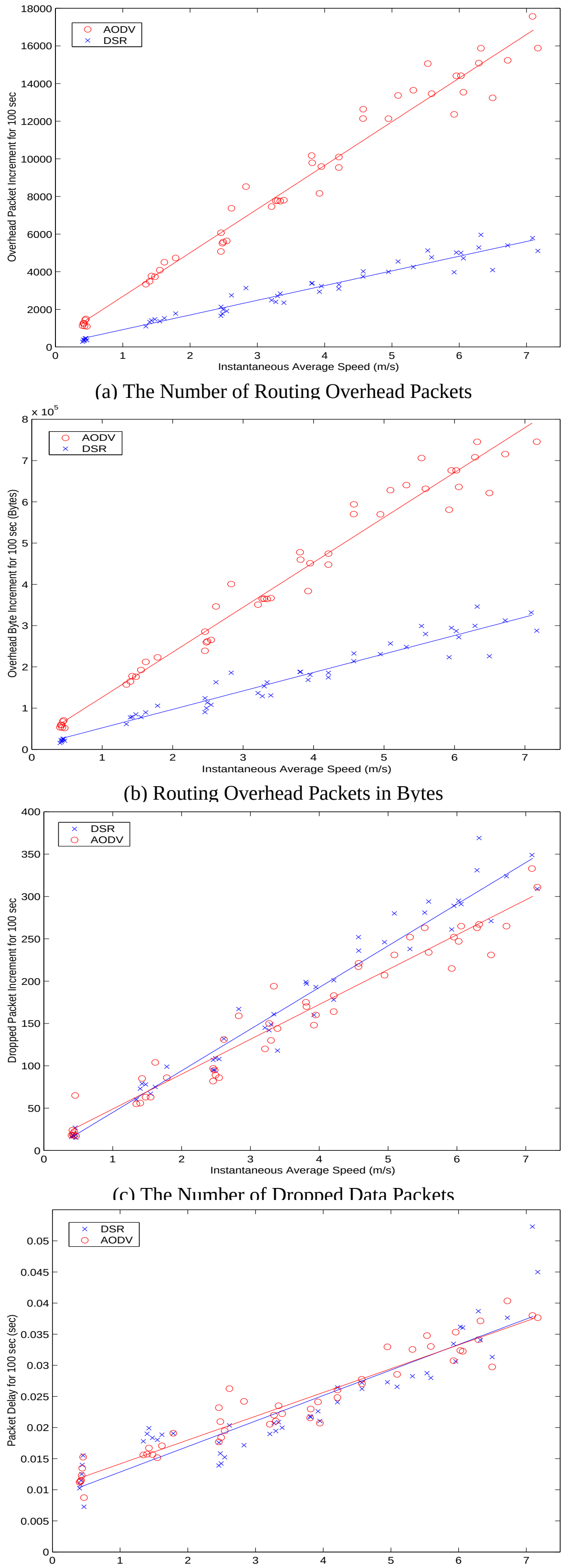


(c) The Number of Dropped Data Packets



(d) Data Packet Delay

Fig. 5. Incremented Metrics for Every 100 Seconds in 900-second Simulation



(d) Data Packet Delay

Fig. 6. Performance Metric vs. Instantaneous Average Node Speed

scenarios of a specific maximum speed. It can be viewed as plotting Fig.4 (average speed vs. simulation time) and Fig.5 (measures vs. simulation time) together in one plot (measures vs. average speed). As shown in Fig.5, it seems reasonable to discard the first 300 seconds as a warm-up period. Therefore, the data before 300 seconds were excluded in obtaining the results shown in Fig.6. All lines in Fig.6 fit the data in a least-squares sense, i.e., minimizing the sum of squared error.

As shown in Fig.6(a) and (b), the routing overhead of DSR and AODV increases linearly as the instantaneous average speed increases. This implies that the ratio of routing overhead between DSR and AODV stays roughly constant even though the actual values of the metric decrease as average speed decreases, which confirms our observation from Fig.5(a) and (b).

Fig.6(c) is consistent with the results in Fig.5(c). As shown in Fig.4, after 300 seconds the instantaneous average node speed of scenarios $[1, 19]$ remains *constant* at 6.1(m/s) . In Fig.6(c), the difference between DSR and AODV at a speed of 6.1(m/s) is about 40. Therefore, the gap between DSR and AODV with speed range $[1, 19]$ in Fig.5(c) is constantly maintained as 40, since the average speed is *stablized* at around 6.1(m/s).

At the same time, Fig.6(c) also shows that the difference between DSR and AODV becomes smaller and smaller as the speed decreases. Below around 2 (m/s) the difference is almost negligible, indicating that DSR and AODV show almost the same performance in terms of the dropped data packet (or equivalently packet delivery ratio) at low speeds. This is why the graphs of speed range $[0, 20]$ in Fig.5(c) seem to converge as time progresses. If the average speed further decreases (i.e., longer simulation duration), the two graphs would finally converge to make them indistinguishable.

In Fig.6(d), there is little difference between DSR and AODV over almost all speed range. This implies that the performance of DSR and AODV in terms of the packet delay is roughly the same. However, note that the data points of DSR seem more deviated from the fitted line than that of AODV. We thus applied a higher order curve to fit the data. A nonlinear fit for data in Fig.6(a)-(c) showed almost no difference from the linear fit, whereas a nonlinear fit for data in Fig.6(d) was slightly different.

Fig.7 is a 3rd order curve fit for the same data shown in Fig.6(d). A distinct feature is that the two performance curves cross twice – at roughly 6.1(m/s) and 1.5(m/s), respectively – within the range of instantaneous average node speed we examined. DSR shows a higher packet delay at relatively higher speeds, and AODV higher at the intermediate speeds and the two almost the same at low

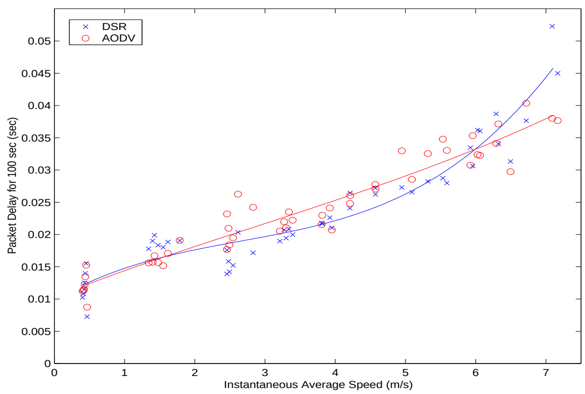


Fig. 7. 3rd Order Curve Fit in Fig.6(d)

speeds. Note the gap in packet delay of DSR and AODV shown in Fig.5(d) with speed range $[1, 19]$ is almost negligible since there is no difference between them at a speed 6.1(m/s) in Fig.7.

IV. Discussion

A. Alternative Improvements

As mentioned before, there could be many potential solutions to avoid the speed decay as time progresses. In the last section we suggested one of the simplest methods by setting a positive minimum speed. Some other improvements can be derived by considering the reason behind speed decay. Slow-moving nodes cause the decay of average node speed and maintain low speed for relatively long times until it reaches the destination. To solve this problem, we can adopt a strategy analogous to the real life: increase the speed if a destination is far away, and reduce the speed when the destination is near. For example, when a destination r and a speed v are chosen and the expected travel time $t = \frac{r}{v}$ exceeds a pre-set threshold $Thresh$, we can reassign the maximum speed to this node. More generally, one may choose to define the (joint) probability distributions of travel distance and the node speed so that the speed is correlated with the destination (distance). Alternatively, since there is only two degrees of freedom (between distance, speed and travel time), one may choose to define a proper distribution for the travel time – one that has finite support – instead of the travel speed.

B. Related Work

Ad hoc network routing protocols have been both extensively and intensively studied in the past few years. The random waypoint model is often used to evaluate the performance of a particular routing protocol.

Broch et al. [2] used the random waypoint model to compare the performance of DSDV, TORA, DSR, and AODV. They chose packet delivery ratio, routing overhead and path optimality as metrics to compare. However, all metrics were reported as time averages over 900 seconds, varying only pause time from experiment to experiment. As we have shown, instantaneous values of these metrics change – some dramatically – over the course of such a simulation.

Perkins et al. [4] performed a similar comparison of DSR and AODV with random waypoint model. They compared the performance observing different metrics such as average packet delay and normalized routing load. However, results were again represented as averages over time.

A recent paper by Perkins, Hughes and Owen [5] shows that node speed, pause time, network size and the number of sources can affect the performance of routing protocols. Surprisingly, node speed is a significant factor, while pause time is not. They used random waypont model but employed Global Mobile System Simulator (GloMoSim) rather than *ns-2*. Again, only time averages were reported. In addition, they only ran simulations for 200 seconds, which is not enough to get past the warm-up period needed.

The Random Waypoint model has also been quite extensively studied. Bettstetter [9] showed by simulation that the random waypoint model does not have a uniform distribution of nodes. Chu and Nikolaidis [6] mathematically showed and confirmed by simulation the same observation. In addition, they showed that there is a relationship between the node distribution and node speed. It was argued that this is partly due to the boundary effect. It was further shown that with different node speed, the mobility scenario poses different connectivity properties. However, such results are again obtained as a time average rather than the change *over* time. The influence of how nodal speed decay influences connectivity properties remains important future work.

In another recent work, Camp, Boleng, and Davies [7] studied and analyzed a variety of mobility models that have been proposed to date, including the random waypoint model. In particular, they showed the change in average percentage (or number) of neighbors as time progresses. Fig.4 in [7] shows that the average number of neighbors has stable mean but increasing variance, which is suggestive of decreased movement of nodes (i.e., nodes are more likely to remain neighbors or remain neighbors for longer periods due to slow movement). But the authors failed to point this out. Various mobility models were also compared in their paper in terms of average speed which is assumed to be the average of minimum and maximum speed rather than the instantaneous node speed.

V. Conclusion

Random waypoint is widely used as a mobility model to compare the performance of various mobile ad hoc network routing protocols. In this paper we have shown that the random waypoint model in its current form fails to reach a steady state in terms of instantaneous average node speed, but rather the speed continuously decreases as simulation progresses. Consequently, this model cannot be used to conduct performance evaluation measured as time averages. Such averages are based on metrics that change over time, sometimes drastically. Considering only these averages can result in misleading or incorrect conclusions.

We showed by an intuitive explanation and a formal study the reason behind the speed decay. Based on our analysis we also proposed a simple solution to the problem, which is to set a positive minimum speed. Our improved model is able to quickly converge to a constant speed. Consequently performance measures resulted from this improvement were also able to stablize. Our analysis, although based on simplifying assumptions, can be used to very accurately estimate the expected instantaneous average node speed given the minimum and maximum speeds and the area of the mobility scenario.

Conceptually the problem revealed in this paper is not a consequence of the random waypoint model itself. In other words, the rationale for the random waypoint model does not limit us from choosing a minimum speed other than zero. For that reason the simple improvement we presented in this paper may still be considered a “random waypoint model”. It is the common belief that one *can* set the minimum speed to zero, and the subsequent wide application of this belief — that made zero the default minimum speed — that has led to this problem and highlight the significance of our work.

Appendix

A. The pdf of distance d from the starting point to the destination

Adopting assumption (A1), the pdf of x and y becomes

$$f_{X,Y}(x,y) = \frac{1}{\pi R_{max}^2}.$$

When we use this analysis to approximate the real simulation scenario where the movement area is defined by a rectangular area of dimension X_{max} and Y_{max}, our results can be used by setting $\pi R_{max}^2 = X_{max}Y_{max}$. In other words, we will approximate the pdf within a rectangular area using a cirle of radius R_{max}:

$$f_{X,Y}(x,y) = \frac{1}{X_{max}Y_{max}} \approx \frac{1}{\pi R_{max}^2}.$$

$$\begin{aligned} P(D \le d) &= \int\int_{\sqrt{x^2+y^2}\le d} f_{X,Y}(x,y)dxdy \\ &= \int_0^{2\pi}\int_0^d \frac{1}{\pi R_{max}^2} r dr d\theta \\ &= \frac{d^2}{R_{max}^2} \end{aligned}$$

$$\begin{aligned} f_D(d) &= \frac{\partial P}{\partial d} \\ &= \frac{2d}{R_{max}^2},\ 0 \le d \le R_{max} \end{aligned}$$

B. The expectation of distance d

$$\begin{aligned} E[D] &= \int_0^{R_{max}} r\frac{2d}{R_{max}^2} dr \\ &= \frac{2}{3}R_{max} \end{aligned}$$

C. The pdf of travel time t

Adopting assumption (A3), speed v is also uniformly distributed from V_{min} to V_{max}. So the pdf of v is

$$f_V(v) = \frac{1}{V_{max} - V_{min}}.$$

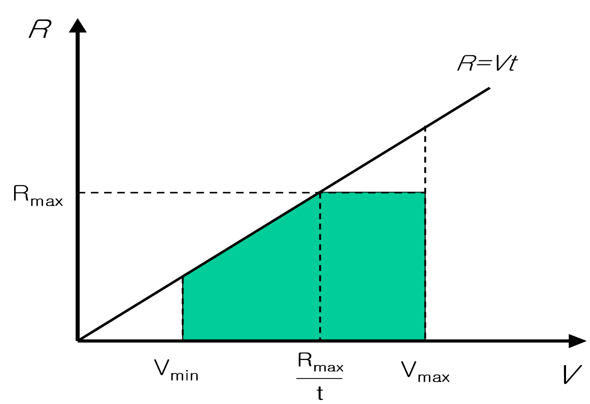


Fig. 8. Distance-Speed Graph

In Fig. 8, the probability $P(T \le t)$ is

$(i)\ R_{max} \le V_{min}t \quad (t > \frac{R_{max}}{V_{min}})$

$$P(T \le t) = P(\frac{R}{V} \le t) = P(R \le Vt) = 0$$

$(ii)\ V_{min}t \leq R_{max} \leq V_{max}t \quad (\frac{R_{max}}{V_{max}} \leq t \leq \frac{R_{max}}{V_{min}})$

$$
\begin{aligned}
P(R \leq Vt) &= \int_{V_{min}}^{\frac{R_{max}}{t}} \int_0^{Vt} f_{R,V}(r,v)drdv \\
&+ \int_{\frac{R_{max}}{t}}^{V_{max}} \int_0^{R_{max}} f_{R,V}(r,v)drdv \\
&= \int_{V_{min}}^{\frac{R_{max}}{t}} \int_0^{Vt} \frac{2r}{R_{max}^2} \cdot \frac{1}{V_{max}-V_{min}} drdv \\
&+ \int_{\frac{R_{max}}{t}}^{V_{max}} \int_0^{R_{max}} \frac{2r}{R_{max}^2} \cdot \frac{1}{V_{max}-V_{min}} drdv \\
&= -\frac{2R_{max}}{3(V_{max}-V_{min})} \cdot \frac{1}{t} \\
&- \frac{V_{min}^3}{3R_{max}^2(V_{max}-V_{min})} \cdot t^2 \\
&+ \frac{V_{max}}{V_{max}-V_{min}}
\end{aligned}
$$

$(iii)\ R_{max} > V_{max}t \quad (t < \frac{R_{max}}{V_{max}})$

$$
\begin{aligned}
P(R \leq Vt) &= \int_{V_{min}}^{V_{max}} \int_0^{Vt} f_{R,V}(r,v)drdv \\
&= \int_{V_{min}}^{V_{max}} \int_0^{Vt} \frac{2r}{R_{max}^2} \cdot \frac{1}{V_{max}-V_{min}} drdv \\
&= \frac{t^2(V_{max}^2+V_{min}^2+V_{max}V_{min})}{3R_{max}^2}.
\end{aligned}
$$

By differentiating $P(T \leq t)$ with respect to t, the pdf of time t becomes

$$
f_T(t) = \begin{cases}
\frac{2(V_{max}^2+V_{min}^2+V_{max}V_{min})}{3R_{max}^2} \cdot t & 0 \leq t \leq \frac{R_{max}}{V_{max}} \\
\frac{2R_{max}}{3(V_{max}-V_{min})} \cdot \frac{1}{t^2} - & \frac{R_{max}}{V_{max}} \leq t \leq \frac{R_{max}}{V_{min}} \\
\frac{2V_{min}^3}{3R_{max}^2(V_{max}-V_{min})} \cdot t & \\
0 & t \geq \frac{R_{max}}{V_{min}}
\end{cases}
$$

D. *The expectation of time t*

$$
\begin{aligned}
E[T] &= \int_0^\infty t \cdot f_T(t)dt \\
&= \int_0^{\frac{R_{max}}{V_{max}}} t \cdot \frac{2(V_{max}^2+V_{min}^2+V_{max}V_{min})}{3R_{max}^2} \cdot tdt \\
&+ \int_{\frac{R_{max}}{V_{max}}}^{\frac{R_{max}}{V_{min}}} t \cdot \frac{2R_{max}}{3(V_{max}-V_{min})} \cdot \frac{1}{t^2} \\
&- t \cdot \frac{2V_{min}^3}{3R_{max}^2(V_{max}-V_{min})} \cdot tdt \\
&= \frac{2R_{max}}{3(V_{max}-V_{min})} \ln\left(\frac{V_{max}}{V_{min}}\right).
\end{aligned}
$$

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