Sparse matrix data structure (one example)

- **Full:**
  - 2-dimensional array of real or complex numbers
  - \((nrows \times ncols)\) memory

- **Sparse:**
  - compressed column storage (CSC)
  - about \((2 \times nzs + ncols)\) memory
Compressed Sparse Row (CSR)
Triples Form

\[
\begin{array}{ccc}
31 & 0 & 53 \\
0 & 59 & 0 \\
41 & 26 & 0 \\
\end{array}
\]

\[
\begin{array}{ccc}
1 & 1 & 31 \\
1 & 3 & 53 \\
2 & 2 & 59 \\
3 & 1 & 41 \\
3 & 2 & 26 \\
\end{array}
\]

About \(3 \times \text{nz}\) memory
Other formats

- List of Lists
  - List of rows, each element is a list of column indices
    - values stored similarly
- Dictionary of Keys
  - (row, column) → value
- Compressed Sparse Blocks
  - Dense matrix of blocks
    - Each block has elements in Morton Z order
  - $A^\top X$ and $A^T \ast X$ takes same time
Graphs and Matrices

- Starting with the matrix:
  - One graph vertex for each row (or column) of the matrix
  - One graph edge \((i,j)\) for each nonzero \(A(j,i)\) in the matrix
  - (Some people point the edges the opposite way, from rows to columns; either way is ok as long as it’s consistent.)

- Or, starting with the graph:
  - The adjacency matrix has \(A(j,i)=1\) if \((i,j)\) is an edge.
• What if you interpret a CSC as a CSR?

• What does a matrix permutation mean?

• What does matrix-vector multiplication mean?
SpMV – Matrix-Vector multiply

\[
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
6 & 7 \\
\end{bmatrix}
\begin{bmatrix}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
7 \\
\end{bmatrix}
= 
\begin{bmatrix}
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\end{bmatrix}
\]
**Breadth First Search - levels**

Min-Plus Semiring:
- addition $\Rightarrow$ min
- multiplication $\Rightarrow$ plus

SpMV over Min-Plus Semiring

Starting vertex has distance zero
Each SpMV finds the next frontier
**Breadth First Search – parents**

Select-Max Semiring:
- addition => max
- multiplication => max

SpMV over Select-Max Semiring

nonzero values in x are starting frontier
value=\text{index}

After SpMV, a nonzero in b contains index of corresponding vertex’ parent
Breadth First Search – parents

1st Frontier:

2nd Frontier: