CS 111: Review Quiz: Answer key

1. 
   \[ A^T = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}, \ A^2 = \begin{pmatrix} 11 & -4 & 2 \\ 2 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}, \ A^T A = \begin{pmatrix} 10 & -3 & 5 \\ -3 & 2 & 0 \\ 5 & 0 & 9 \end{pmatrix}. \]

2. \[ ||(3, 1, 4, 1, 5)^T||_2 = \sqrt{52} \approx 7.2111 \]

3. 
   \[
   \begin{pmatrix} 2 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 4 \end{pmatrix}
   \]

4. 
   \[ x = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \]

5. There are many answers to this. Here’s one: In Matlab notation, take \( A = [1, 2; 2, 4] \) and \( b = [3; 3] \). Explanation 1 (column view): Matrix \( A \) is singular, so the space spanned by its columns is only one-dimensional, and it consists of multiples of the vector \([1; 2] \), which do not include \( b \). Explanation 2 (row view): The two lines described by the row form of \( Ax = b \) are parallel and hence do not intersect. Explanation 3 (brute force view): No matter what \( x \) is, the second entry of \( Ax \) will be equal to twice the first entry of \( Ax \), which rules out \( b \).

6. There are many answers to this. Here’s one: In Matlab notation, take \( A = [1, 2; 2, 4] \) and \( b = [3; 6] \). Two solutions are \( x = [1; 1] \) and \( x = [3; 0] \).

7. No, it’s not possible to have exactly two solutions to \( Ax = b \). If \( x \) and \( y \) are two different solutions, then there are infinitely many solutions: \( x + \alpha(y - x) \) is a solution for every \( \alpha \).

8. \( A \) has two eigenvalues, 3 and 5. Any multiple of \([1; 1] \) is an eigenvector corresponding to 3, and any multiple of \([1; -1] \) is an eigenvector corresponding to 5.

9. \( f'(x) = 21x^2 - 4x + 4 \).

10. \( \partial z/\partial x = e^{y/2} \), and \( \partial z/\partial y = (x/2)e^{y/2} \).

11. \( f(x) = x^3/3 - \cos x + c \) for some constant \( c \) (any constant will do).

12. The height is maximum when the derivative \( dh/dt \) is zero. \( dh/dt = 1280 - 32t \), which is zero when \( t = 40 \), at which time the height is \( h = 25600 \) feet. The bullet hits the ground when \( h = 0 \), which means \( 1280t = 16t^2 \), which means \( t = 1280/16 = 80 \) seconds after firing. (The other solution to \( h = 0 \) is of course \( t = 0 \).)

13. \( y = e^{x^2/2} \).