1. Review of Counting

Ordering matters?	Repetition allowed? Yes	No
Yes	n^k	P(n,k)
No	C(n + k – 1,k)	C(n,k)

(a) How many 3 letter strings can I make out of the word QUESTION?
Order matters? YES
Repetition Allowed? NO
Answer: $P(10, 3)$

(b) I have 10 fingernails and 25 different bottles of nail polish. How many different ways can I paint my nails?
Order matters? YES
Repetition Allowed? YES
Answer: 25^10 (Each nail has 25 different possibilities/.)

(c) A test has 10 questions and you are required to answer 6 of them. How many different ways can you select the 6 questions?
Order matters? NO - The teacher will not be able to tell in what order you chose the questions after you turn your test in.
Repetition Allowed? NO
Answer: $C(10, 6)$

(d) A donut shop has 20 different kinds of donuts. Assuming there are at least a dozen of each kind, how many different ways can you choose a dozen donuts?
Order matters? NO
Repetition Allowed? YES
Answer: $C(20 + 12 - 1, 12)$

2. Prove for sets A and B:
If $\mathcal{P}(\mathcal{A}) \subseteq \mathcal{P}(\mathcal{B})$ then $A - B = \emptyset$.

If we let X be the statement $\mathcal{P}(\mathcal{A}) \subseteq \mathcal{P}(\mathcal{B})$ and Y be the statement $A - B = \emptyset$, we know that $X \to Y$ is equivalent to the contrapositive $\neg Y \to \neg X$.

$\neg Y$ is $A - B \neq \emptyset$
$\neg X$ is $\mathcal{P}(\mathcal{A}) \not\subseteq \mathcal{P}(\mathcal{B})$

So we will prove $A - B \neq \emptyset \to \mathcal{P}(\mathcal{A}) \not\subseteq \mathcal{P}(\mathcal{B})$.

If $A - B \neq \emptyset$ then there is some x such that $x \in A$ and $x \notin B$. Therefore $\{x\} \subseteq A$ and $\{x\} \not\subseteq B$, or equivalently, $\{x\} \in \mathcal{P}(\mathcal{A})$ and $\{x\} \notin \mathcal{P}(\mathcal{B})$. Therefore $\mathcal{P}(\mathcal{A}) \not\subseteq \mathcal{P}(\mathcal{B})$.