29. Show that for all positive integers m and n,

$$n\binom{m+n}{m} = (m+1)\binom{m+n}{m+1}$$

First, convert into factorial form

$$\frac{n(m+n)!}{m!((m+n)-m)!} = \frac{(m+1)(m+n)!}{(m+1)!((m+n)-(m+1))!}$$

Simplify the denominators

$$\frac{n(m+n)!}{m!n!} = \frac{(m+1)(m+n)!}{(m+1)!(n-1)!}$$

Then, multiply left side by $\frac{1}{n}/\frac{1}{n}$

$$\frac{\frac{1}{n}n(m+n)!}{\frac{1}{n}m!n!} = \frac{(m+1)(m+n)!}{(m+1)!(n-1)!}$$

Simplify left side (note $\frac{n!}{n} = (n-1)!$)

$$\frac{(m+n)!}{m!(n-1)!} = \frac{(m+1)(m+n)!}{(m+1)!(n-1)!}$$

Multiply right side by $\frac{1}{m+1}/\frac{1}{m+1}$

$$\frac{(m+n)!}{m!(n-1)!} = \frac{\frac{1}{m+1}(m+1)(m+n)!}{\frac{1}{m+1}(m+1)!(n-1)!}$$

Simplify right side (note $\frac{(m+1)!}{m+1} = m!$)

$$\frac{(m+n)!}{m!(n-1)!} = \frac{(m+n)!}{m!(n-1)!}$$

Done!