1. Answer True or False for each of the following:

   (a) \( O(n) + O(\log n) + O(\sqrt{n}) = O(n) \).
   (b) \( \sum_{i=1}^{\sqrt{n}} i = \Theta(n) \).

2. We use a random hash function to map \( n \) keys into a table of size \( m \), where \( m \) is an even integer. What is the probability that no key hashes into an \textbf{odd-numbered} location?

3. Prove Markov’s inequality. If \( X \) is a non-negative random variable, and \( a > 0 \), then \( \Pr(X \geq a) \leq E(X)/a \).

4. Show the following regarding the maximum key in a binary min-heap with \( n \) nodes.

   (a) It must be at one of the leaves.
   (b) There are \( \lceil n/2 \rceil \) leaves.
   (c) Any algorithm to find the maximum must take \( \Theta(n) \) time.

5. Suppose we have a \( d \)-heap with \( N \) keys initially. We perform \( M \) \textsc{PercolateUp} and \( N \) \textsc{PercolateDown} operations on it.

   (a) What is the total running time for all these operations in terms of \( N, M \) and \( d \)?
   (b) What is the running time if \( d = 2 \).
   (c) What is the running time if \( d = \Theta(N) \)?
   (d) What choice of \( d \) achieves the minimum total running time?