Data Structures and Algorithms (CS 130b)	**Prof. Suri**
Exercises 3	
Handed Out: Feb 24	*Due: Mar 5*

1. (10 pts) Recall that a clique is a collection of mutually adjacent vertices in a graph. The decision version of the CLIQUE problem takes as input a graph G and an integer k, and decides if G has a clique of size k or not. The optimization version of the problem takes as input a graph G and returns the largest clique in G. Show that the CLIQUE problem is self-reducible: that is, if the decision version has a polynomial time algorithm, then the optimization version also has a polynomial time algorithm.

2. (10 pts) Consider the following problem. The input is an undirected graph G and an integer k. The problem is to decide if G contains a clique of size k AND an independent set of size k. Show that this problem is NP-complete.

3. (10 pts) The input to the 3-COLORING problem is an undirected graph G and the problem is to decide if the vertices of G can be colored using 3 colors such that no two adjacent vertices receive the same color. In the 4-COLORING problem, we need to decide if the vertices can be colored in this way using at most 4 colors. Show by reduction that if the 4-COLORING problem has a polynomial algorithm, then so does the 3-COLORING problem.

4. (10 pts) Let $\mathcal{U}$ be a finite set, and let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ be a collection of subsets of $\mathcal{U}$. Given an integer k, the SET-COVER problem asks if there is a sub-collection of k sets $\mathcal{S}' \subset \mathcal{S}$ whose union covers all the elements of $\mathcal{U}$. That is, $|\mathcal{S}'| = k$, and $\bigcup_{S_i \in \mathcal{S}'} S_i = \mathcal{U}$. Prove that SET-COVER is NP-complete.

5. (10 pts) Suppose you are managing a communication network, modeled by a directed graph $G = (V, E)$. There are m users and each user i wants to *reserve* a path P_i in the network to transmit his data. A set of path can be *feasibly reserved* if no two of them share any node in common.

   The *Path Selection Problem* is the following: given a directed graph $G = (V, E)$, a set of path requests $P_1, P_2, \ldots, P_m$, and a positive integer k, is it possible to feasibly reserve at least k of these paths?

   Prove that the Path Selection Problem is NP-complete.