

CS130B, WINTER 2009, GREEDY PRACTICE PROBLEMS

1. Consider the following problem:

INPUT: A set $S = \{(x_i, y_i) | 1 \leq i \leq n\}$ of intervals over the real line.

OUTPUT: A maximum cardinality subset A of S such that no pair of intervals in A overlap.

Consider the following algorithm:

1. Repeat until S is empty.
2. Select the interval I that overlaps the least number of other intervals.
3. Add I to the initial solution set A .
4. Remove all intervals from S that overlap with I .

Prove or disprove that this algorithm solves the problem.

2. Consider the Change Problem in Austria. The input to this problem is an integer L . The output should be the minimum cardinality collection of coins required to make L shillings of change (that is, you want to use as few coins as possible).

In Austria the coins are worth 1, 5, 10, 20, 25, 50 Shillings. Assume that you have an unlimited number of coins of each type. Formally prove or disprove that the greedy algorithm (that takes as many coins as possible from the highest denominations) correctly solves the Change Problem.

So for example, to make change for 234 Shillings the greedy algorithms would take four 50 shilling coins, one 25 shilling coin, one 5 shilling coin, and four 1 shilling coins.

3. The input consists of n skiers with heights p_1, \dots, p_n , and n skies with heights s_1, \dots, s_n . The problem is to assign each skier a ski to minimize the AVERAGE DIFFERENCE between the height of a skier and his/her assigned ski. That is, if the skier i is given the ski a_i , then you want to minimize:

$$\sum_{i=1}^n (|p_i - s_{a_i}|) / n$$

(a) Consider the following greedy algorithm. Find the skier and ski whose height difference is minimized. Assign this skier this ski. Repeat the process until every skier has a ski. Prove or disprove that this algorithm is correct.

(b) Consider another greedy algorithm. Give the shortest skier the shortest ski, give the second shortest skier the second shortest ski, give the third shortest skier the third shortest ski, etc. Prove or disprove that this algorithm is correct.

HINT: One of the above greedy algorithms is correct and one is incorrect for the other.

4. You are given an ordered list of n words. The length of the i th word is w_i , that is the i th word takes up w_i spaces. (For simplicity assume that there are no spaces between words.) Your goal is to break this ordered list of words into lines; this is called a layout. Note that you can not reorder the words.

The length of a line is the sum of the lengths of the words on that line. The ideal line length is L . No line may be longer than L , although it may be shorter. The penalty for having a line of length K is $L - K$. The total penalty of a layout is the sum of the line penalties. Your problem is to design a layout that minimizes the total penalty. Prove or disprove that the following greedy algorithm correctly solves this problem:

For $i = 1$ to n Place the i th word on the current line if it fits else place the i th word on a new line