Please typeset your answers, or write very clearly. If TAs cannot read your handwriting, they will not grade your assignment.

1. (20 pts) Show the Huffman tree and the corresponding codes for the following distribution of punctuation characters and digits (their frequencies are shown in parentheses):

   colon (100), space (605), newline (100), comma (705), 0 (431), 1 (242), 2 (176), 3 (59), 4 (185), 5 (250), 6 (174), 7 (199), 8 (205), 9 (217).

2. (20 pts) Suppose a server has \( n \) customers waiting to be served. The service time required by each customer is known in advance: \( t_i \) minutes for customer \( i \). For instance, if the customers are serviced in the order of increasing index \( i \), then the \( i \)th customer has to wait \( \sum_{j=1}^{i} t_j \) minutes. Our goal is to produce an order that minimizes the total waiting time:

   \[
   T = \sum_{i=1}^{n} (\text{time spent waiting by customer } i)
   \]

   Give an efficient algorithm for computing the optimal order in which to service the customers. You must prove that the algorithm always finds optimal order, and also analyze its worst-case running time.

3. (10 pts) Suppose we want to construct Huffman “ternary codes,” where each “bit” can have three values: 0, 1, 2. In other words, the tree representing the code is ternary; each node has up to three children, labeled 0, 1, and 2. Show an optimal ternary code for text that contains 4 letters with frequencies 0.1, 0.2, 0.2, 0.5. Based on this example, how would you modify Huffman’s algorithm so that it produces optimal ternary codes?