1. (25 pts) Consider a long quiet country road with \( n \) houses scattered along it. We can picture the road as a long line segment, with an eastern endpoint and a western endpoint. We want to place cell phone towers at certain points along the road so that every house is within 2 miles of one of the base stations.

Given the location of the \( n \) houses along the road, design an efficient algorithm for computing the minimum number of base stations. Prove the correctness of your algorithm, and analyze its time complexity.

2. (25 pts) Determine an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers:

\[
a : 1, \quad b : 1, \quad c : 2, \quad d : 3, \quad e : 5, \quad f : 8, \quad g : 13, \quad h : 21
\]

(The notation \( x : n \) stands for letter \( x \) and its frequency \( n \).)

(a) Draw the Huffman code tree, and write down the codeword for each letter.

(b) Next, generalize your answer to find the optimal code when the frequencies are the first \( n \) Fibonacci numbers.

3. (25 pts) Prove the following two properties of the Huffman encoding scheme.

- If some character occurs with frequency strictly more than \( \frac{2}{5} \), then there is guaranteed to be a codeword of length 1.
- If all the characters occur with frequency strictly less than \( \frac{1}{3} \), then no codeword of length 1 will be produced.

4. (25 pts) Design an \( O(n) \) time algorithm to solve the following problem, which occurs in program analysis. We are given a set of \( n \) variables \( x_1, x_2, \ldots, x_n \), and two sets of constraints: equality constraints of the form “\( x_i = x_j \)” and disequality constraints of the form “\( x_i \neq x_j \)”. That is, each equality constraint demands that a pair of variables have the same value, and each disequality demands that a pair of variables must have different values. (The total number of constraints is at most \( n \).)

Given an instance of this problem, your task is to decide if all constraints can be satisfied. Describe your algorithm, prove its correctness, and analyze its time complexity.
Submission Instructions.

- Submissions and grading will be done through Gradescope: http://gradescope.com
- Sign up for the course using the entry code: 9ZYR5W
- After signup, you can see the homeworks for the course and due date in your account. Select Homework 1 and just follow the instructions on the website. You can either submit one image per question, or upload one pdf and identify the pages for each question.