1. (15 pts) Let $A$, $B$, and $C$ be three strings each $n$ characters long. We want to compute the longest subsequence that is common to all three strings. Give a polynomial time algorithm for this problem. Derive the dynamic programming recurrence, clearly explain what each term stands for, and justify the correctness of the recurrence. Analyze the worst-case time complexity of your algorithm.

2. (15 pts) Recall that a clique is a collection of mutually adjacent vertices in a graph. The decision version of the CLIQUE problem takes as input a graph $G$ and an integer $k$, and decides if $G$ has a clique of size $k$ or not. The optimization version of the problem takes as input a graph $G$ and returns the largest clique (not just its size) in $G$. Show that the CLIQUE problem is self-reducible: that is, if the decision version has a polynomial time algorithm, then the optimization version also has a polynomial time algorithm.

3. (15 pts) Let $\mathcal{U}$ be a finite set, and let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ be a collection of subsets of $\mathcal{U}$. Given an integer $k$, the SET-COVER problem asks if there is a sub-collection of $k$ sets $S' \subset \mathcal{S}$ whose union covers all the elements of $\mathcal{U}$. That is, $|S'| = k$, and $\bigcup_{S_i \in S'} S_i = \mathcal{U}$. Prove that SET-COVER is NP-complete.

4. (15 pts) We have a set of $n$ transmitters in a region and set of available frequencies. We label the entire frequency set as $\{1, 2, \ldots, m\}$, and each transmitter $i$ is assigned a subset $F_i \subset \{1, 2, \ldots, m\}$. (Different transmitters have different assigned sets.) The transmitter $i$ can only transmit on a frequency in $F_i$. In addition, there is a set of interfering transmitter pairs, which cannot transmit at the same frequency because they interfere. For instance, if $(i, j)$ is an interfering pair with $F_i = \{1, 5, 8\}$ and $F_j = \{2, 5, 12, 20\}$, then $i$ and $j$ both cannot be allocated frequency 5. A frequency allocation is valid if each transmitter $i$ gets a frequency from its assigned set $F_i$ and no two interfering transmitters get the same frequency. Prove that the frequency allocation problem is NP-complete.