

Self Assessment for Advanced Algorithms

If you are considering taking this course, the following self-assessment may help you decide if this is the right course for you. You should be able to solve this problems comfortably.

1. The health department thinks that upto m out of n , where $m < n$, water reservoirs in a certain city have become contaminated. It is expensive to conduct a water test, and the city, facing a budget crisis, wants to minimize its expense. A test can be conducted on several water samples simultaneously by mixing small portions of these samples—a positive test outcome then tells us if at least one of the samples is contaminated. Describe an algorithm that identifies all the contaminated reservoirs using $O(m \log_2 n)$ tests.
2. Given a set S of n points in the plane, define a **linear clique** to be a subset $X \subseteq S$ whose points all lie on a line. We want to determine the largest linear clique in the set. Either show that the problem is NP-Complete, or describe a polynomial time algorithm.
3. In a society, each couple bears children until they have their first son, and then they stop having children. Suppose that there is equal likelihood (50%) of the newborn being a boy or a girl. What is expected number of children in each family?
4. Given an undirected graph $G = (V, E)$, and an integer k , the **cycle-elimination** problem is to decide if all the cycles of the graph can be eliminated by deleting at most k edges from the graph. Either show that the problem is NP-complete, or describe a polynomial-time algorithm for it.
5. You are a prisoner in a foreign land, and your fate will be determined by a little game. There are two jars, one with 50 white marbles, and one with 50 black marbles. At this point, you are allowed to redistribute the marbles however you wish: the only requirement is that after you are done with the redistribution, every marble must be in one of the two jars.

Afterwards, both jars will be shaken up, and you will be blindfolded and presented with one of the jars at random. Then you pick one marble out of the jar given to you. If the marble you pull out is white, you live; if black, you die.

How should you redistribute the marbles to maximize the probability that you live. What is this maximum probability (roughly)?