1. (25 pts) Indy-506 is a car race that takes place on an infinite straight-line track. There are \( n \) cars, each starting at a different position along the track, and each traveling at a different, but constant, speed during the race. (Figure below is an illustration, where the vector lengths are used to show different speeds of the cars.)

We are interested in a particular car \( G \), driven by Dr. Geometry. At any time during the race, the car in front of \( G \) is called the leader of \( G \). The leader of \( G \) changes when one of the following three events happen: (a) \( G \) overtakes its leader; (b) \( G \) is overtaken by another car; or (c) the leader of \( G \) takes over its own leader.

What is the maximum number of times that the leader of \( G \) can change during the entire race? Give the proof of your bound.

2. (25 pts) Recall that Chan’s optimal convex hull algorithm “guesses” the convex hull size \( h \), by using a parameter \( m \), which increases as follows: \( m = \min(2^{2^t}, n) \), for \( t = 1, 2, \ldots \). Re-analyze Chan’s algorithm using the following two alternative ways to increase \( m \), and derive the worst-case bounds for his algorithm for these variants:

- \( m = \min(2^t, n) \), for \( t = 1, 2, \ldots \)
- \( m = \min(2^{2^t}, n) \), for \( t = 1, 2, \ldots \)

3. (25 pts) Let \( \{a_1, a_2, \ldots, a_n\} \) be a set of \( n \) distinct symbols. Call a string \( S \) consisting of these symbols a \( DS_2 \) sequence if (1) two consecutive symbols are always distinct, and (2) two symbols don’t alternate more than twice (hence the subscript 2). In other words, the sequence does not contain either of the following two patterns: (i) \( \cdots a_i a_i \cdots \) (ii) \( \cdots a_i \cdots a_j \cdots a_i \cdots a_j \cdots \)

Show that the maximum possible length of a \( DS_2 \) string is \( 2n - 1 \).

**Remark:** In our plane sweep algorithm for the Voronoi Diagram, if you think of each site’s identity as a symbol, then the parabolic front, written out as a sequence of arcs, is a \( DS_2 \) sequence.
4. (25 pts) Consider a set \( S = \{s_1, s_2, \ldots, s_n\} \) of \( n \) non-intersecting line segments in the plane. Define a segment \( s_i \) to be above \( s_j \) if at some \( x \)-coordinate \( s_i \) is above \( s_j \). (I.e. there is a vertical line intersecting both \( s_i \) and \( s_j \), and the intersection point with \( s_i \) is higher.) The above relation is a partial order; if \( s_i \) and \( s_j \) don’t overlap in their \( x \)-span, then neither is above the other. Figure below shows an example of 5 line segments, with arrows indicating the ”above” relationship among some of the pairs.

Prove that above relation is **acyclic**, meaning that if we follow the above pointers, we can never return to the same segment.