Binomial (min) Heaps

([W] Section 6.8. Binomial Queues ARE SIMILAR TO Binomial HEAPS)

Binomial Heaps allow for EFFICIENT MERGING.

Binomial (min) HEAP: SEQUENCE OF BINOMIAL TREES (THAT SATISFY THE MIN HEAP PROPERTY) OF DIFFERENT ORDER.

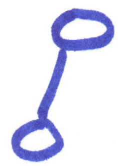
BINOMIAL TREE OF ORDER K ($K \geq 0$)

- $K=0$: The TREE IS A SINGLE NODE
- $K>0$: THE TREE HAS A ROOT WHOSE CHILDREN ARE ROOTS OF BINOMIAL TREES OF ORDERS $K-1, K-2, \ldots, 1, 0$.

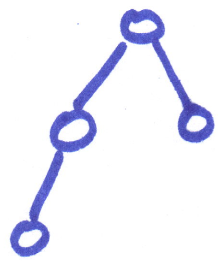
BINOMIAL TREES

$k=0$

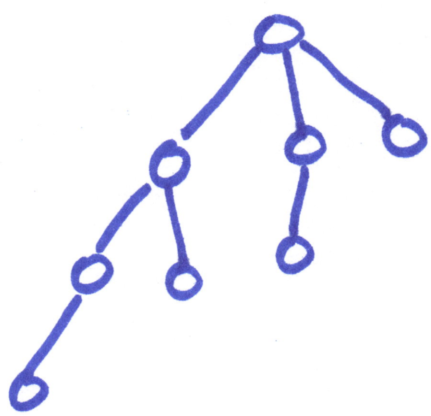
$k=1$



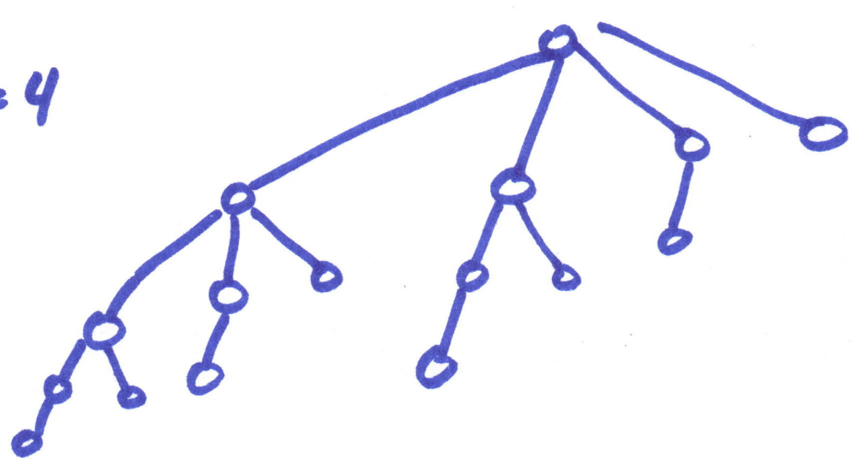
$k=2$



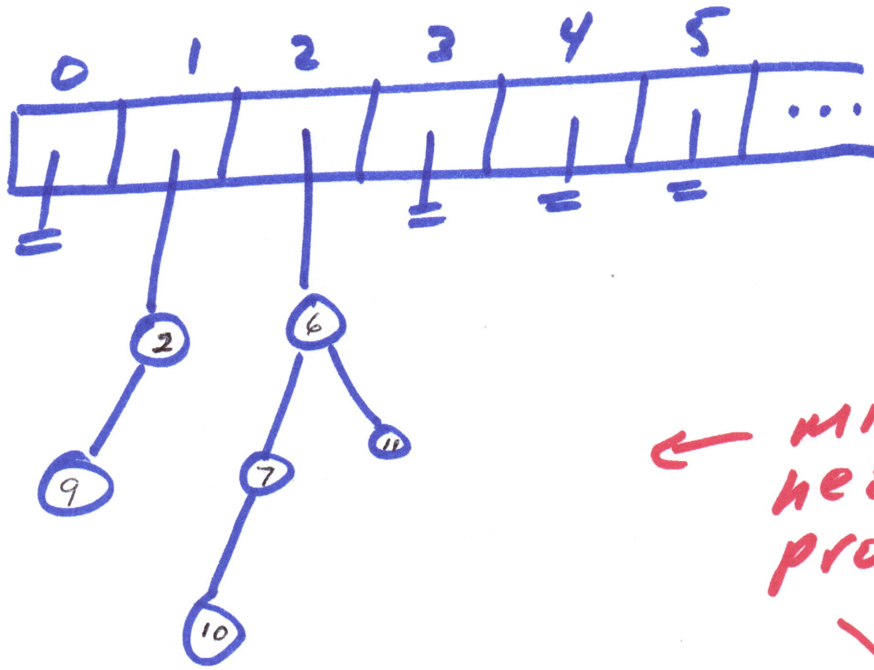
$k=3$



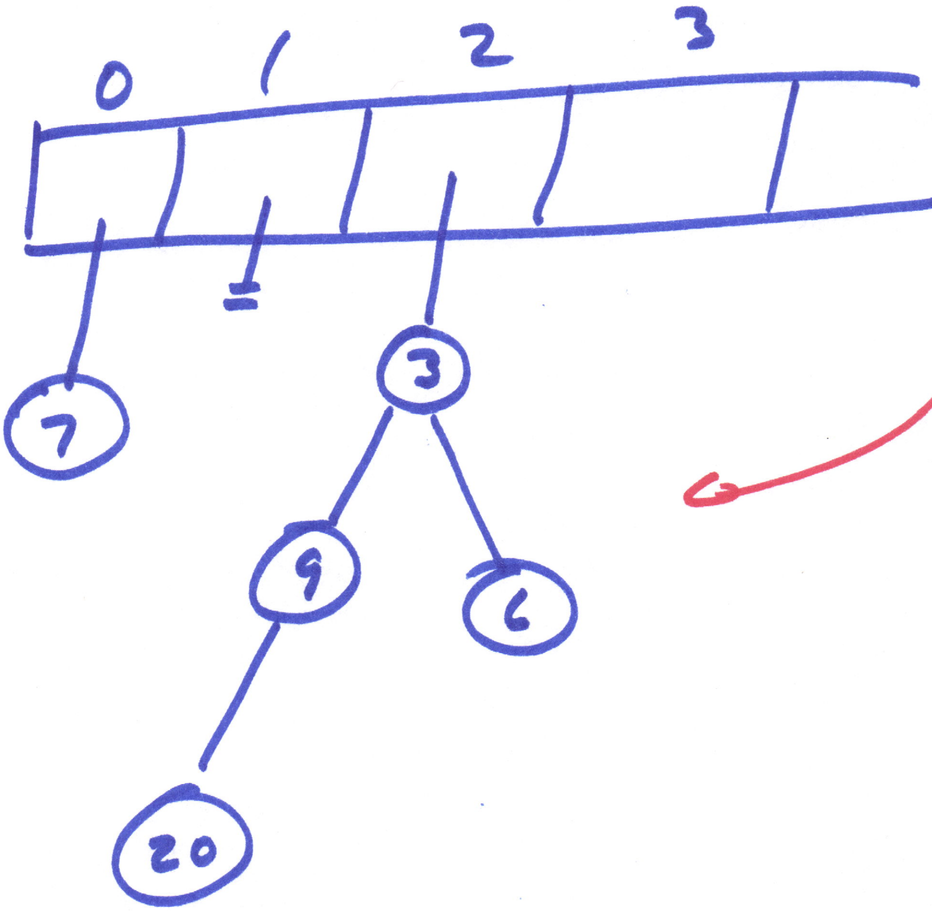
$k=4$



BINOMIAL HEAP



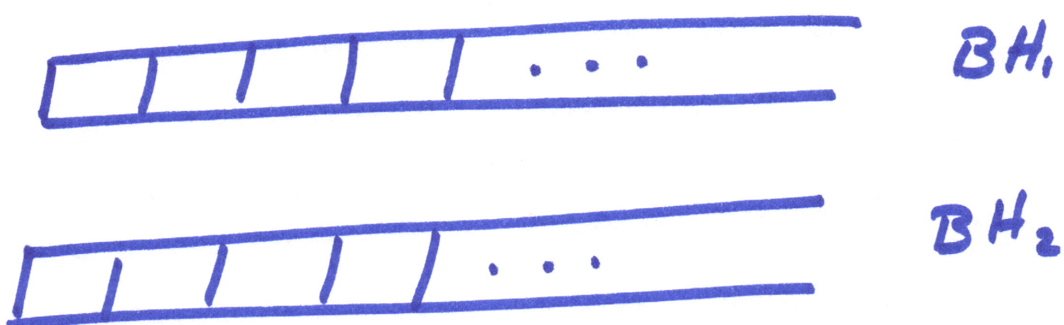
min heap property



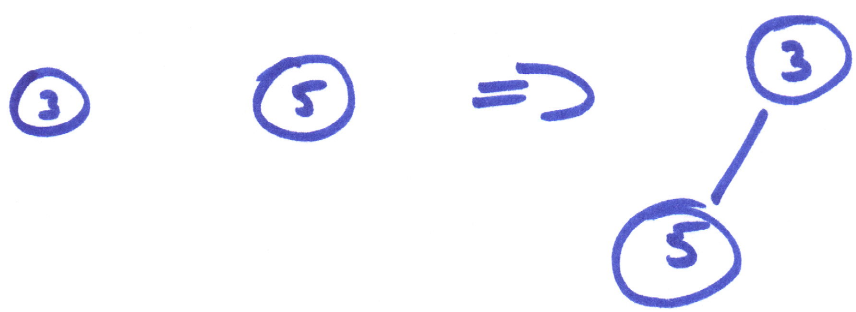
A BINOMIAL TREE OF ORDER k HAS 2^k NODES & HEIGHT k.

IMPLEMENTATION

MERGE

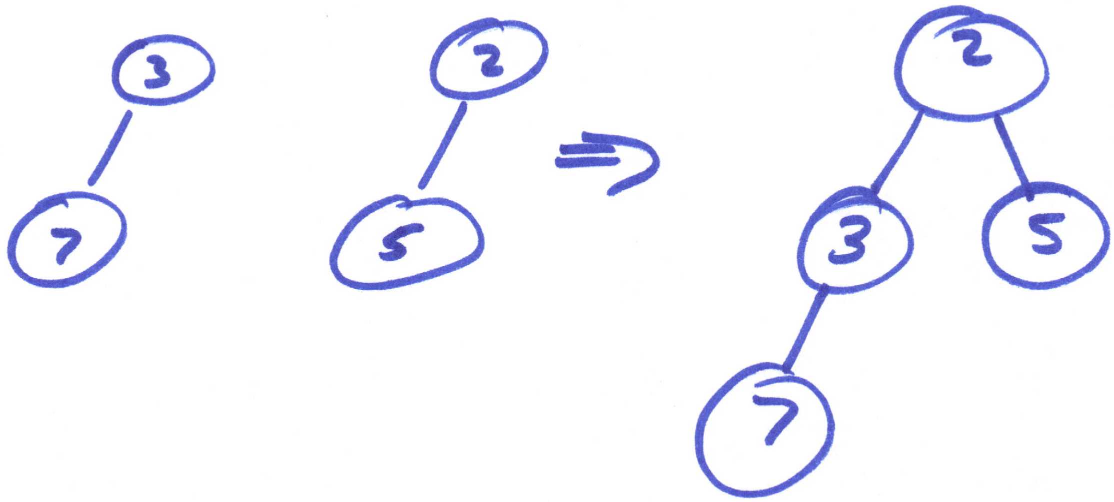


MERGE BINOMIAL TREES OF SAME ORDER:

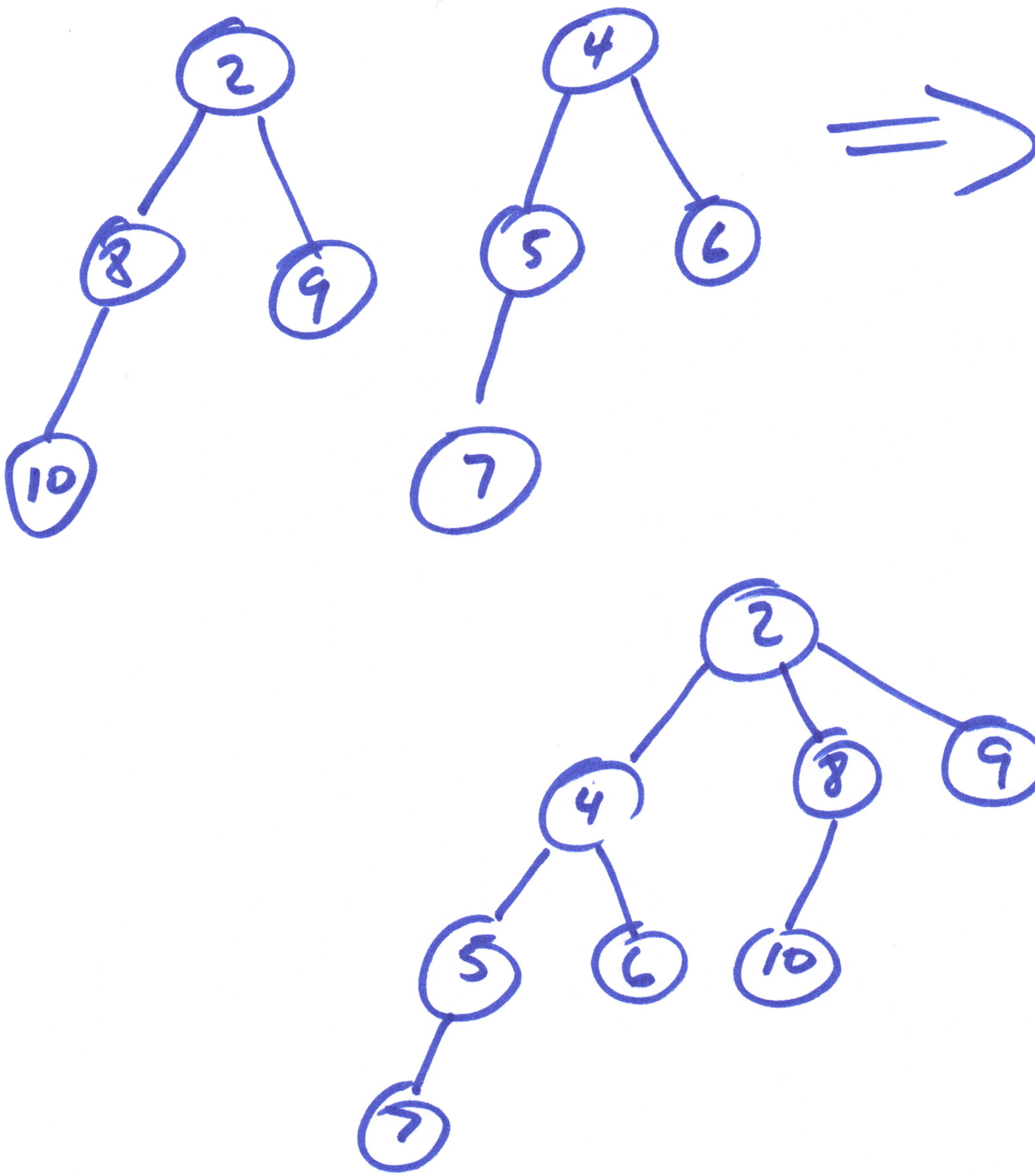


k = 0

k=1



k=2



MERGE: BH_1 + BH_2 ⟹ BH

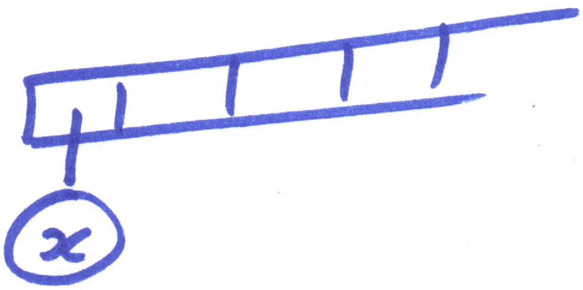
CARRY = Null
for i=0, ... size of array
LET J BE THE NUMBER OF NOT NULL POINTERS IN $BH_1[i]$, $BH_2[i]$ & CARRY.

CASE
:J=3: MERGE CARRY + $BH_1[i]$ INTO CARRY
$BH[i] \leftarrow BH_2[i]$
:J=2: MERGE BOTH TREES INTO CARRY
$BH[i] \leftarrow$ Null
:J=1: LET $BH[i]$ GET THE NON-NULL POINTER + set carry ← Null
:J=0; $BH[i] \leftarrow$ Null

endfor
if CARRY ≠ NULL then error

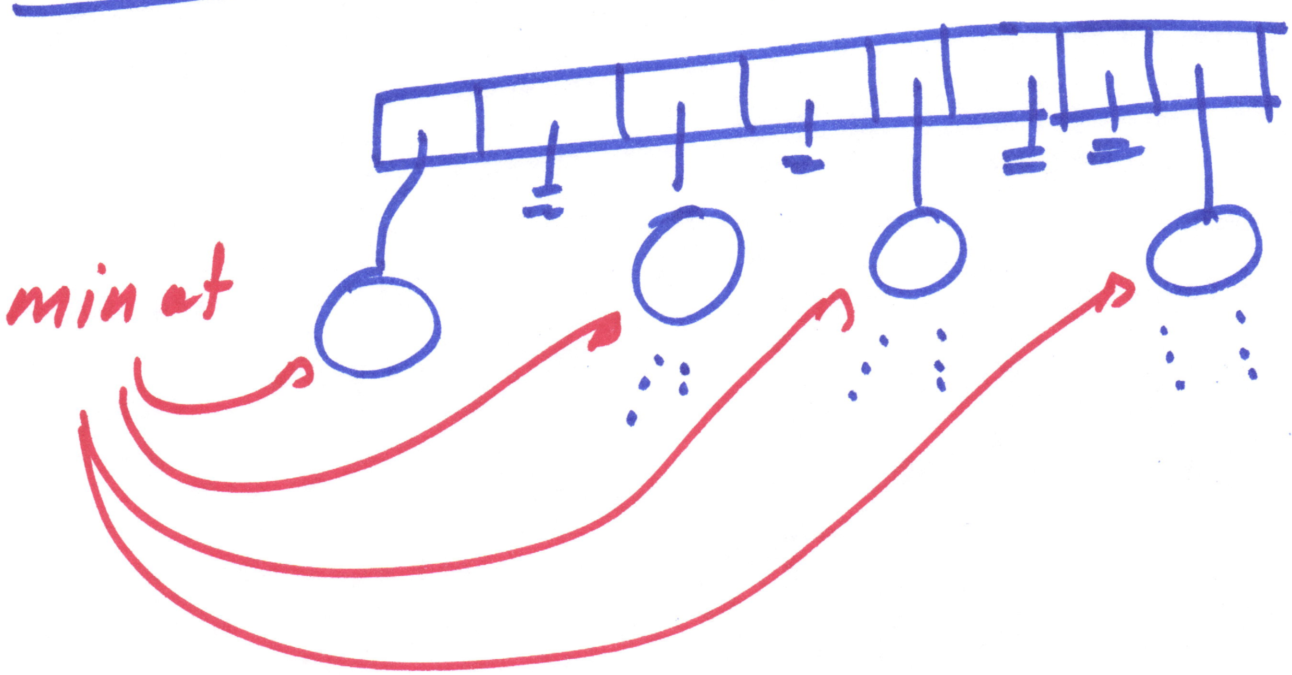
INSERT x in BH_1

Create Binomial HEAP BH_2 with only x



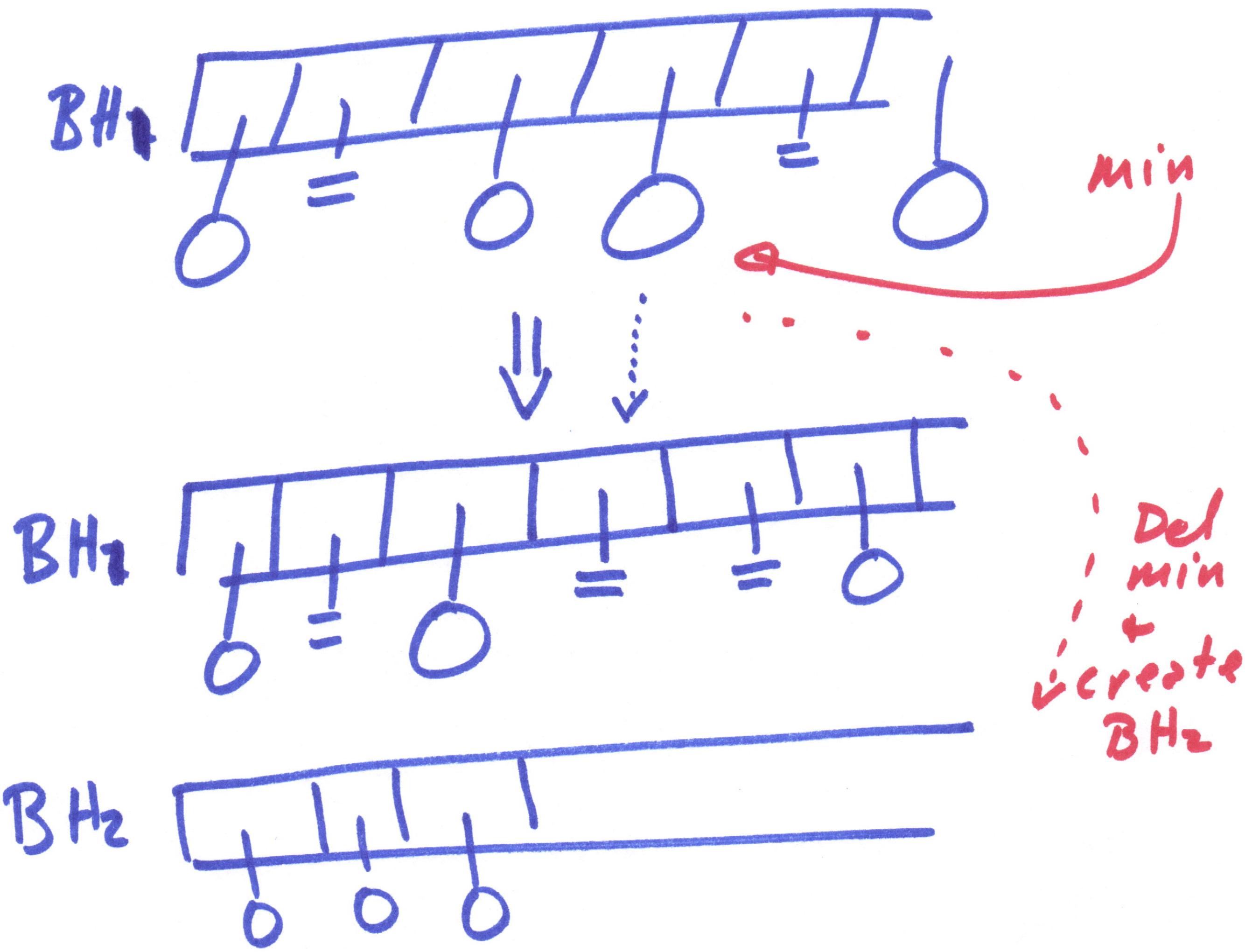
MERGE BH_1 & BH_2

FIND MIN BH



Delete Min BH.

FIRST LOCATE MIN

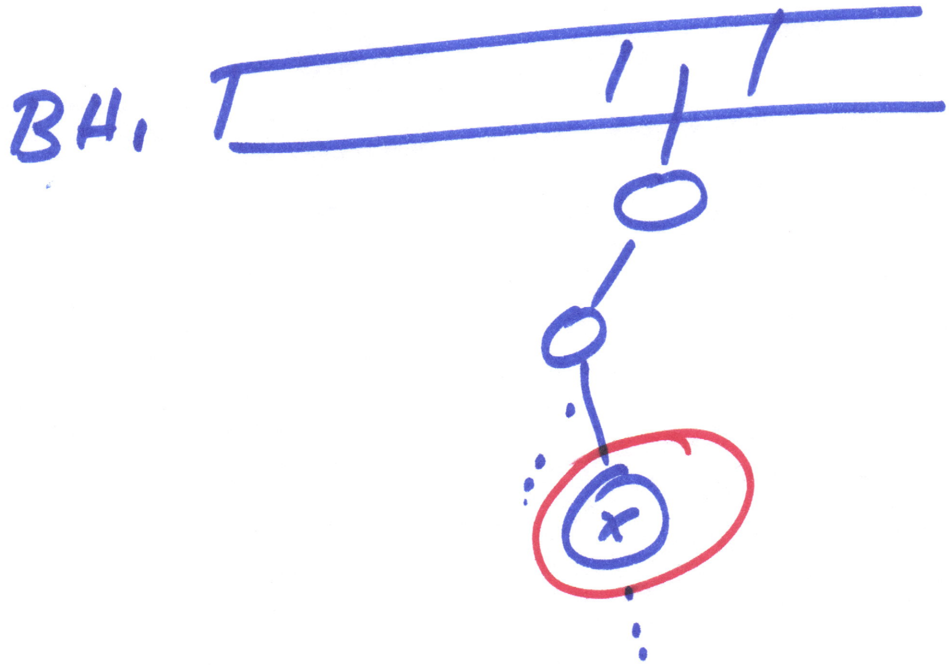


$\Downarrow$

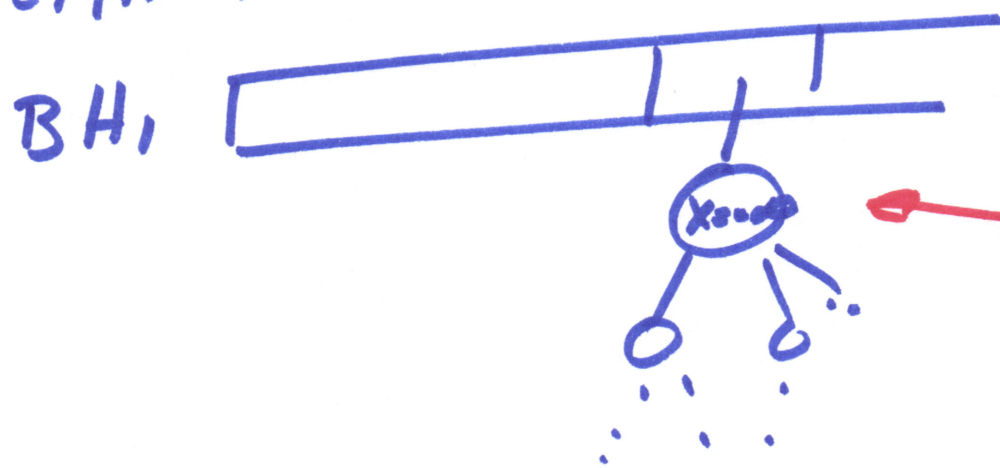
MERGE BH_1 + BH_2

Delete x from BH_1

LOCATE x



CHANGE VALUE TO $-\infty$



Delete

⇑

op like
Delete min