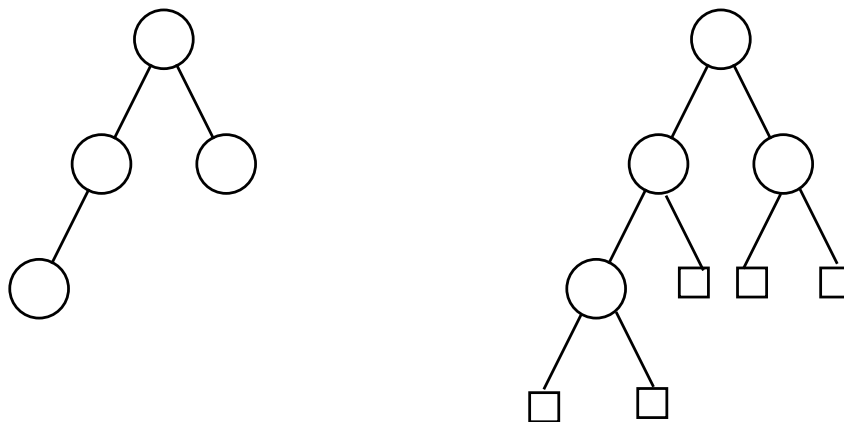


## Leftist Trees

- Linked binary trees.
- Insert and DeleteMin (or Delete Max) takes  $O(\log n)$  time.
- Can Meld (Merge) two leftist trees in  $O(\log n)$  time.

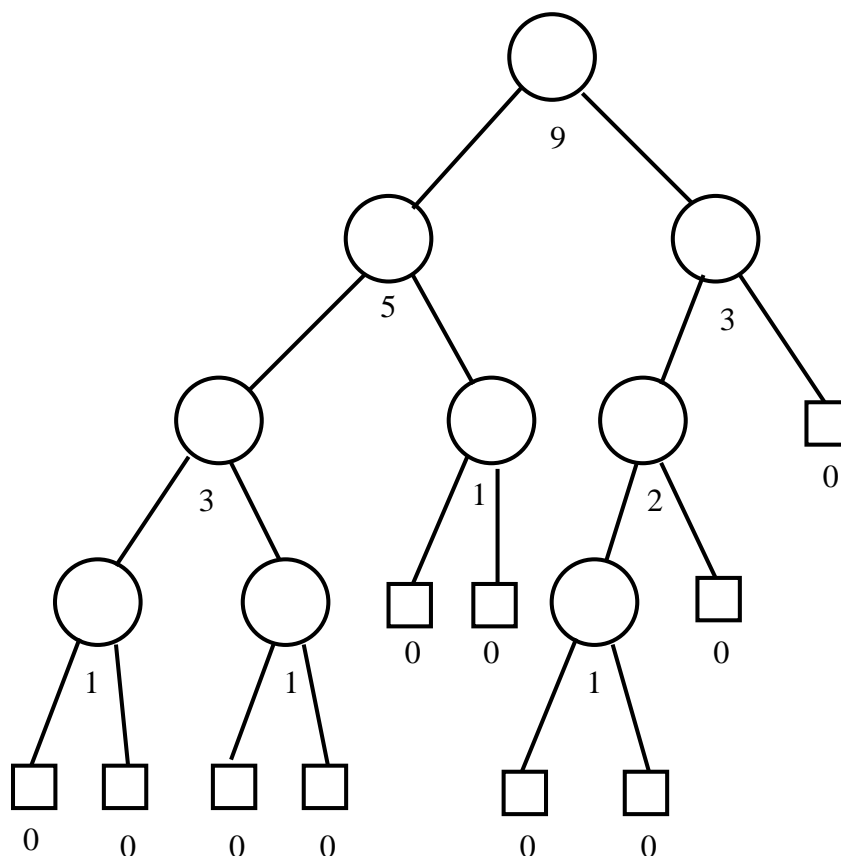
## Extended Binary Trees

(Add external nodes)



## $W(\ )$ Weight Function

$W(x)$ : For any node  $x$ ,  $W(x)$  is the total number of (internal) nodes in the subtree rooted at  $x$  (including  $x$ ).



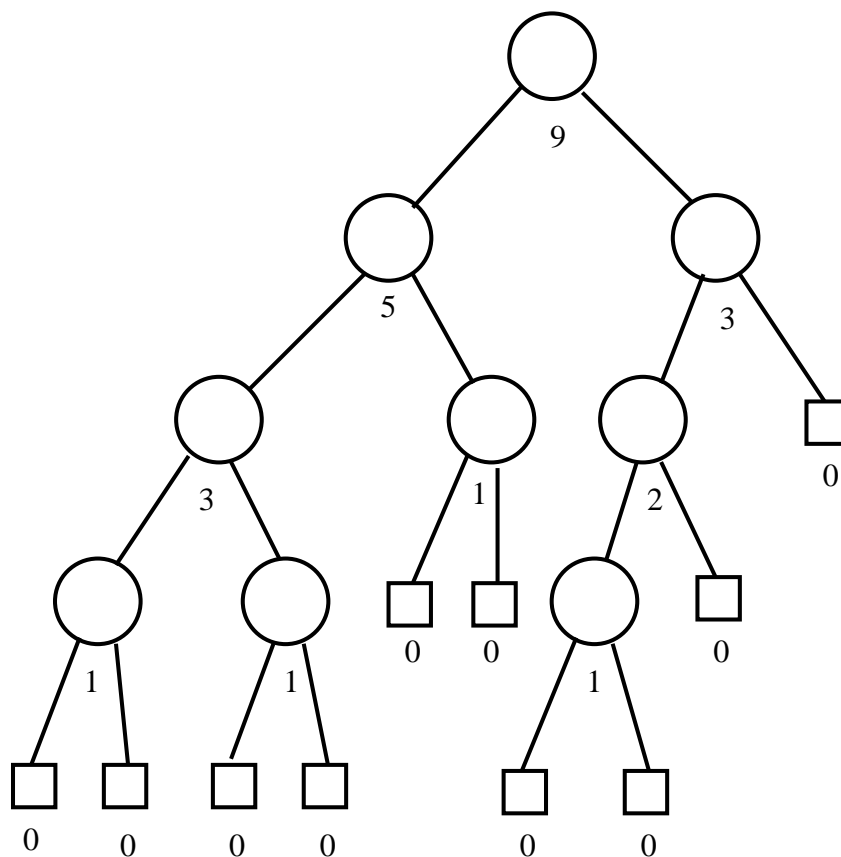
## Computing $W(x)$

$$S(x) = \begin{cases} 0 & \text{if } x \text{ is an external node} \\ W(lc(x)) + W(rc(x)) + 1 & o.w. \end{cases}$$

where lc (rc) represents leftchild (rightchild).

## Weight-Biased Leftist Trees (WBLT)

- A Binary tree is a WBLT
- iff
- for every internal node  $x$ ,  
 $W(lc(x)) \geq W(rc(x))$

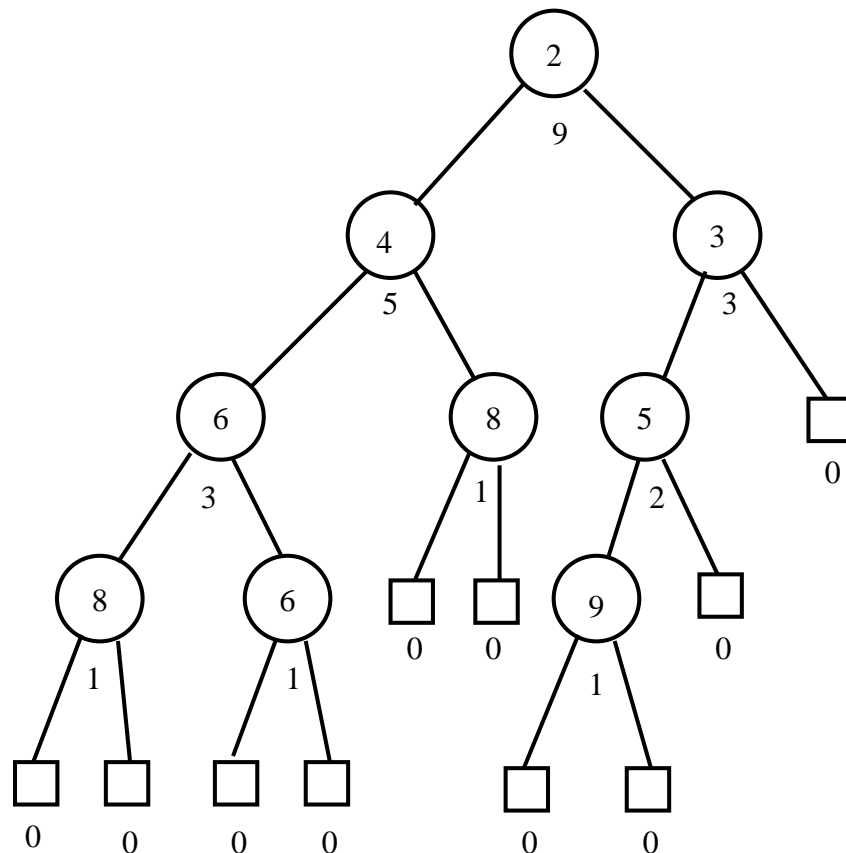


## Property of WBLTs

- A shortest root to external node path has length  $O(\log W(\textit{Root}))$ .
- The rightmost path has this length.

## Min WBLTs

A Min WBLT that satisfies the “Min Heap ordering” is a Min WBLT.

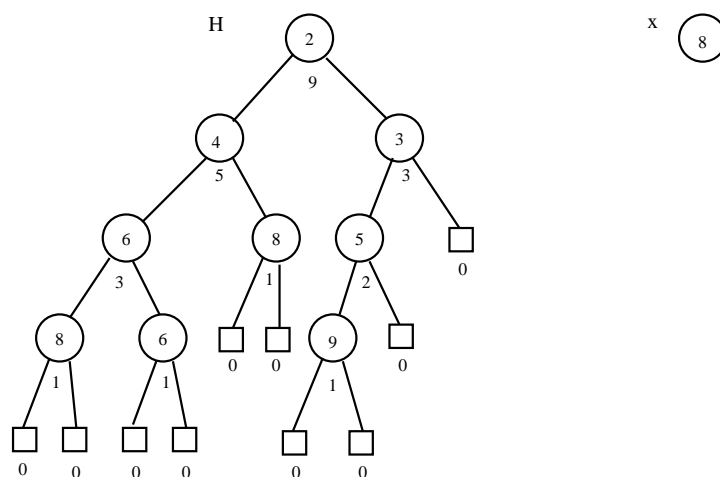


The Insert, DeleteMin and Meld operations can be performed in  $O(\log n)$  time.

## Insert Operation

Insert  $x$  in WBLT  $H$  is just MELD( $x, H$ )

Insert x with value 8 => Meld H and the single node WBLT x with value 8

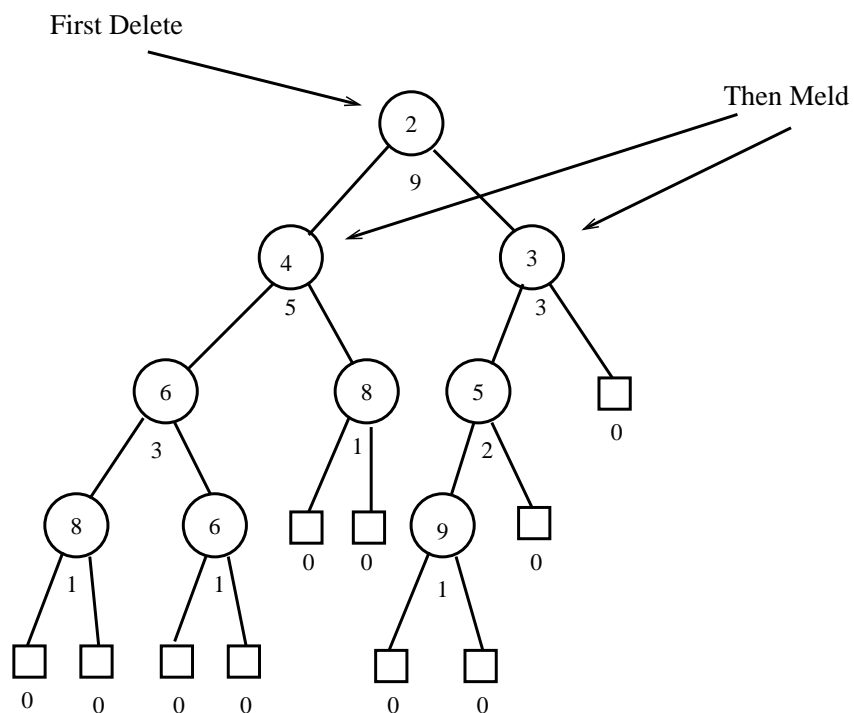




## DeleteMin Operation

DeleteMin from WBLT  $H$  is just  
 $\text{MELD}(lc(H), rc(H))$

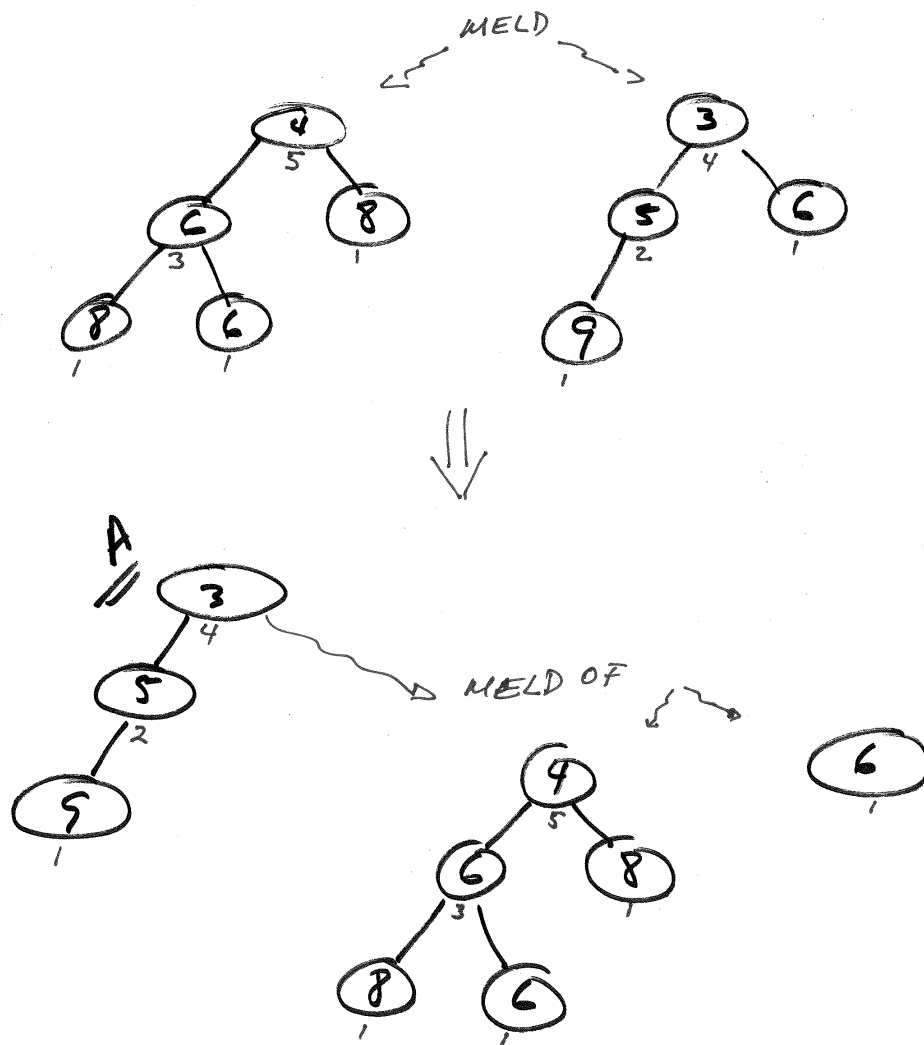
Delete Min  $\Rightarrow \text{Merge}(lc(\text{Root}), rc(\text{root}))$



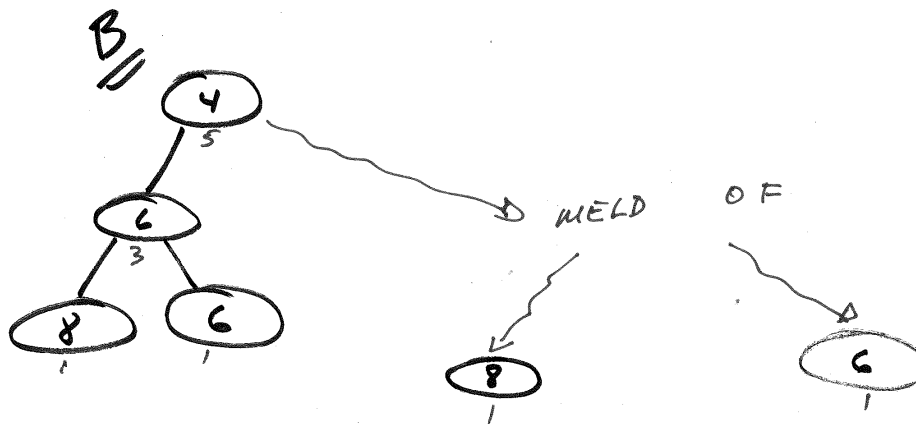
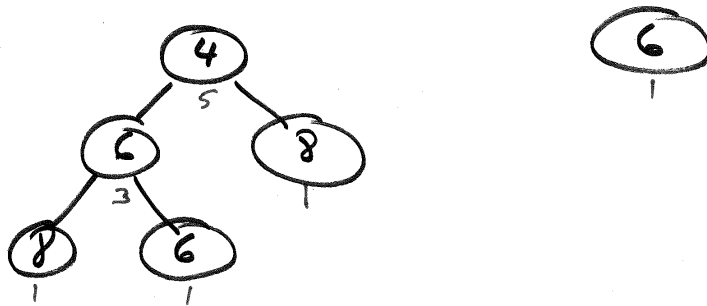
## Meld Two WBLTs

Traverse rightmost paths. See example beginning next page.

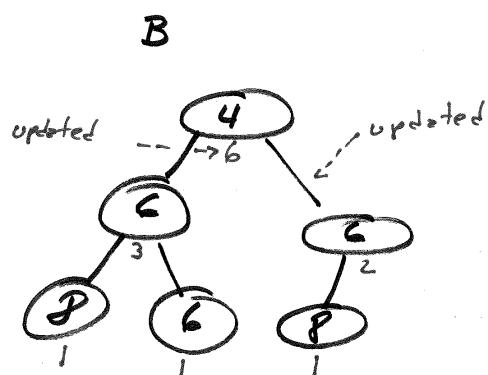
## MELD TWO WBLT



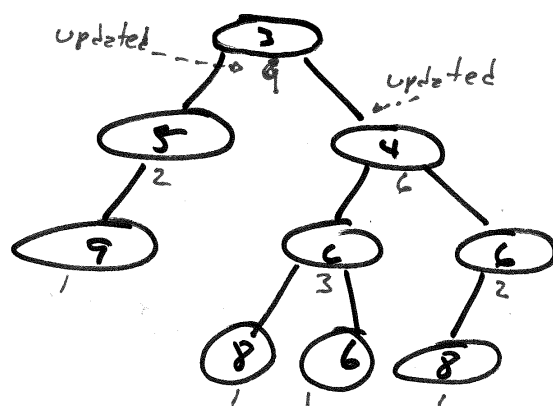
MELD OF



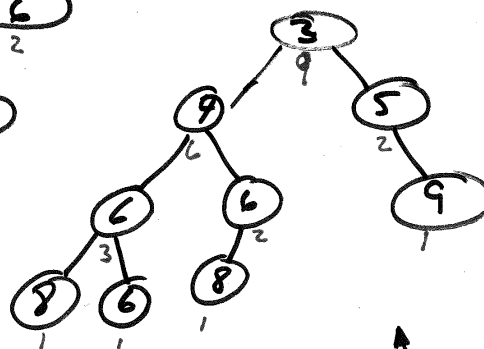
PASTE BACK IN B



PASTE BACK IN A



Swap Lc & rc



Resulting WBLT