Homework 2

Posted: Monday, April 9, 2018 – 11:59pm
Due: Wednesday, April 18, 2018 – 2pm (Gradescope – instruction will follow)

Task 1 – DFAs (6 points)

Consider the DFA $M$ with alphabet $\Sigma = \{a, b\}$ described by the following transition graph:

![DFA Diagram]

a) Represent the DFA as a tuple $M = (Q, \Sigma, \delta, q_0, F)$. In particular, describe $Q$, $\delta$, and $F$.

b) What is $\delta^*(q_0, ab)$? What is $\delta^*(q_0, b^{20}aab)$? What is $\delta^*(q_0, aba^{34}b^{123})$?

c) Describe the language $L(M)$ accepted by $M$.

Hint: Any description is accepted, but it may be easier to represent $L(M)$ as the union of more than one language.

Task 2 – Regular Languages (8 points)

Show that each of the following languages with alphabet $\Sigma = \{0, 1\}$ is regular. Concretely, give corresponding DFAs accepting them, and justify (in words, but as precisely as possible) why your DFAs accept the corresponding languages.

a) $L_1 = \emptyset$

b) $L_2 = \Sigma^*$

c) $L_3 = \{\lambda\}$

d) $L_4 = \{01^30^i : i \geq 1\}$

e) $L_5 = \{u011v : u, v \in \{0, 1\}^*\}$

f) $L_6 = \{00w : w \in \Sigma^*, w \text{ does not contain two consecutive 1's}\}$

g) $L_7 = \{w \in \Sigma^* : |w| \text{ is a multiple of 3}\}$

h) $L_8 = \{w \in \Sigma^* : w \text{ contains an even number of 0's}\}$
**Task 3 – Finite Languages**

The goal of this task is to convince yourself that all finite languages (i.e., languages that only contain a finite number of sentences) are regular.

a) Let \( L = \{\lambda, a, b, ba, bbb\} \) a language with alphabet \( \Sigma = \{a, b\} \). Show that \( L \) is regular by giving a corresponding DFA.

b) Show that every finite language \( L \) is regular.

**Hint:** Given any \( \Sigma \) and any finite language \( L \subseteq \Sigma^* \), explain a method to construct the corresponding DFA \( M \) such that \( L(M) = L \). Take inspiration from what you did in a).

**Task 4 – NFAs**

Throughout this task, consider the NFA \( M = (Q, \Sigma, \delta, q_0, F) \) with alphabet \( \Sigma = \{0, 1\} \) defined by the following transition graph.

![NFA Transition Graph]

a) Complete the formal description of \( M \), i.e., give \( Q, \delta \) and \( F \) matching the above transition graph. Recall that here \( \delta : Q \times (\Sigma \cup \{\lambda\}) \to 2^Q \), i.e., \( \delta \) returns a subset of \( Q \), which could be empty.

b) What is \( \delta^*(q_0, 0) \)? \( \delta^*(q_0, \lambda) \)? \( \delta^*(q_0, 0110) \)?

c) Describe the language \( L = L(M) \) accepted by the NFA \( M \).

**Task 5 – Simplified NFAs**

Show that for every NFA \( M \), there exists an NFA \( M' \) such that \( L(M) = L(M') \), and moreover, \( M' \) has a single final state which is different from its initial state.

**Hint:** How you would reduce the number of final states in the NFA from Task 4 without affecting the language accepted? How can you transform this idea to work with any NFA with multiple final states?