Homework 3

Posted: Friday, Oct 9, 2015 – 11:59pm
Due: Tuesday, Oct 20, 2015 – 3pm (HFH 2108) or 3:30pm (in class)

Task 1 – From NFAs to DFAs

Transform the following two NFAs with alphabet \( \Sigma = \{a, b\} \) into equivalent DFAs. Use the general transformation method presented in class. Present and explain each individual step explicitly, in addition to giving the final transition graph.

a) 

b) 

Note: Do not minimize the resulting DFA, except for removing unreachable states.

Task 2 – DFA Minimization

Consider the DFA \( M \) defined by the following transition graph.

a) Which states are unreachable in \( M \), i.e., there is no way an execution of \( M \) will ever reach these states?
b) Let $M'$ be the DFA obtained from $M$ by removing the unreachable states found in a). Which pairs of states are indistinguishable in $M'$, and which ones are not?

**Hint:** You can either list all pairs, or give equivalence classes consisting of mutually indistinguishable states. You do not need to use any specific algorithm from the textbook, the task can be solved directly by inspecting the DFA.

c) Use the findings from b) to minimize the number of states in $M'$. Explicitly give the resulting DFA $M''$.

d) Alice claims she has found a four-state DFA for the language $L = \{a^ib^j : i \geq 2, j \geq 3\}$. Is she right? Justify your answer in detail!

**Task 3 – Regular Expressions**

(8 points)

Throughout this task, let us fix the alphabet $\Sigma = \{a, b, c\}$.

a) Use set notation to describe the languages $L(r_1), \ldots, L(r_6)$ defined by the following regular expressions. (We usually omit $\cdot$ for concatenation.)

- $r_1 = \lambda cc^*$
- $r_2 = (\emptyset + c)(\emptyset + c)(\emptyset + c)$
- $r_3 = (\lambda + c)(\lambda + c)(\lambda + c)$
- $r_4 = \emptyset \lambda$
- $r_5 = \emptyset + \lambda$
- $r_6 = \emptyset c^*$

b) Give a regular expression which defines the language of all strings in $\Sigma^*$ containing the substring $aba$ or the substring $cbabc$.

c) Give a regular expression describing the language accepted by the following NFA.

![NFA Diagram]

**Task 4 – Closure Properties**

(6 points)

Let $L_1$ and $L_2$ be regular languages. Show that the following languages are also regular.

a) The difference $L_1 \setminus L_2 = \{w \in L_1 : w \notin L_2\}$.

b) The symmetric difference $L_1 \oplus L_2 = (L_1 \setminus L_2) \cup (L_2 \setminus L_1)$.

c) The reversal $L_1^R = \{w^R : w \in L_1\}$.

**Hint:** You can use the closure properties presented in class for union, intersection, $\cdot$, concatenation, or the fact that every regular language has a DFA/NFA. Try to find the shortest answer.