Homework 3

Task 1 – From NFAs to DFAs (12 points)

Transform the following three NFAs with alphabet $\Sigma = \{0, 1\}$ into equivalent DFAs. Use the general transformation method presented in class. Present and explain each individual step explicitly, in addition to giving the final transition graph.

a) 

b) 

c) 

Note: Do not minimize/modify the resulting DFA, except for removing unreachable states.

Task 2 – Regular Expressions (6 points)

Throughout this task, let us fix the alphabet $\Sigma = \{a, b, c\}$.

a) Use set notation to describe the languages $L(r_1), \ldots, L(r_6)$ defined by the following regular expressions. (We usually omit $\cdot$ for concatenation.)

- $r_1 = \lambda aa^*$
- $r_2 = (\emptyset + a)(\emptyset + b)(\emptyset + c)$
- $r_3 = (\lambda + a)(\lambda + b)(\lambda + c)$
- $r_4 = \emptyset + \lambda$
- $r_5 = (a + b + c + \emptyset)^*$
- $r_6 = \lambda^*$

b) Give a regular expression which defines the language of all strings in $\Sigma^*$ containing the substring $bbaa$ or the substring $cba$.

Task 3 – From NFAs to Regular Expressions (4 points)

Give a regular expression describing the language accepted by the following NFA.
Task 4 – Closure Properties  

Let $L_1$ and $L_2$ be regular languages. Show that the following languages are also regular.

a) The difference $L_1 \setminus L_2 = \{w \in L_1 : w \not\in L_2\}$.

b) The symmetric difference $L_1 \oplus L_2 = (L_1 \setminus L_2) \cup (L_2 \setminus L_1)$.

c) The reversal $L_1^R = \{w^R : w \in L_1\}$.

**Hint:** You can use the closure properties presented in class for union, intersection, $\star$, concatenation, or the fact that every regular language has a DFA/NFA. Try to find the shortest answer.