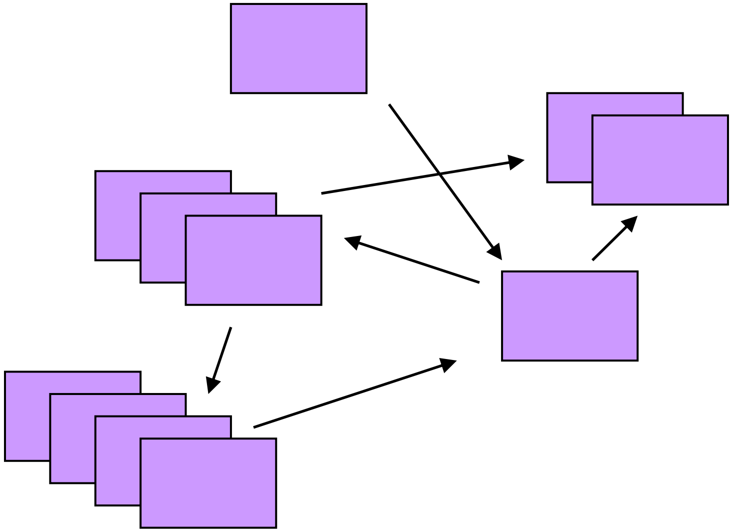
Case Study: Matrix Computation in Web Search and Mining



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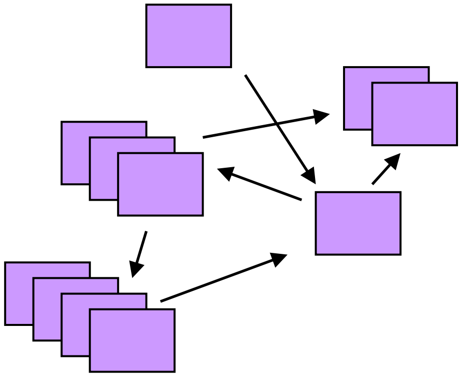
Some slides are from C. Li (UCI) an R. Mooney (UTexas)

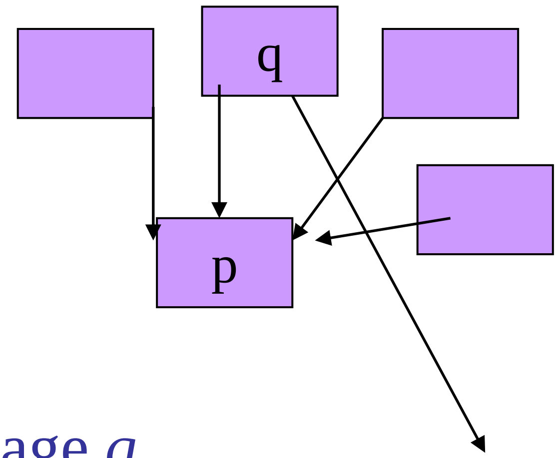
Mining Web Graph for Search Ranking



- PageRank Algorithm by Google
- Intuition:
  - The importance of each page is decided by what other pages "say" about this page
  - One naïve implementation: count the # of pages pointing to each page (i.e., # of inlinks)
- Problem:
  - We can easily fool this technique by generating many dummy pages that point to a page

Initial PageRank Idea

- Rank r(p) for page *p*:

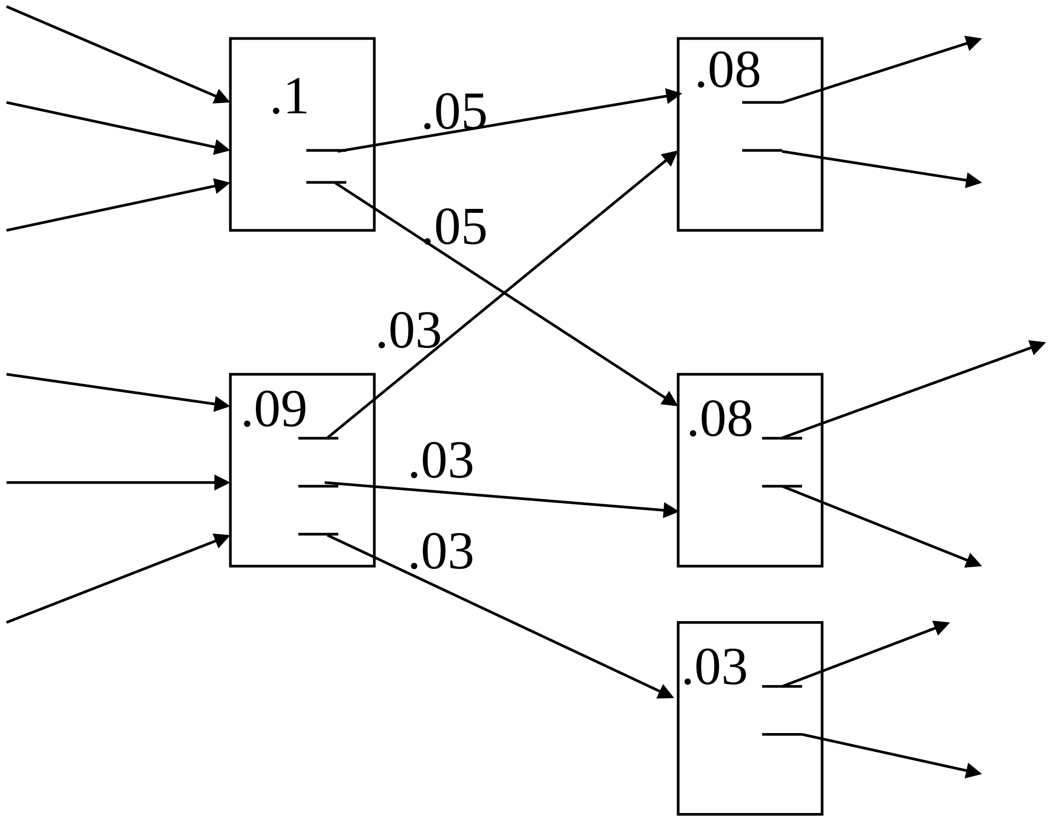


$$r(p) = c \sum_{q:q \to p} \frac{r(q)}{N_q}$$

  - N_q is the total number of out-links from page *q*.
  - A page, *q*, “gives” an equal fraction of its authority to all the pages it points to (e.g. *p*).
  - *c* is a normalizing constant set so that the rank of all pages always sums to 1.
- Rank of a page represents its authority on the web

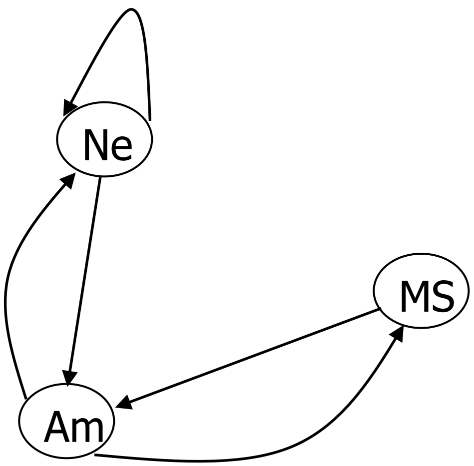
Initial PageRank Idea (cont.)

- Can view it as a process of PageRank "flowing" among pages.



Representing PageRank with Matrix Computation

- Assume three web sites: Netscape, Amazon, and Microsoft.
- Their weights are represented as a vector



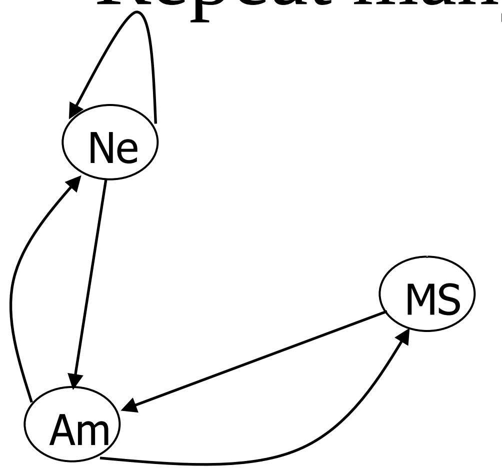
$$\begin{bmatrix} n \\ m \\ a \end{bmatrix} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 \\ 1/2 & 1 & 0 \end{pmatrix} \begin{bmatrix} n \\ m \\ a \end{bmatrix}$$

For instance, in each iteration, half of the weight of AM goes to NE, and half goes to MS.

How to solve the matrix equations?

Iterative method

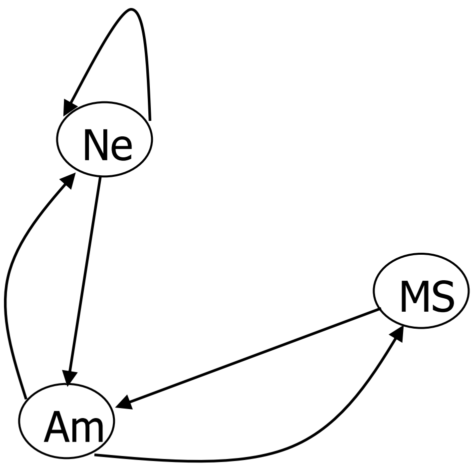
- Initially all rank values are 1
- Compute the new values (n,m,a) using their old values from the equations
- Repeat many iterations



$$\begin{bmatrix} n \\ m \\ a \end{bmatrix}_{new} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 \\ 1/2 & 1 & 0 \end{pmatrix} \begin{bmatrix} n \\ m \\ a \end{bmatrix}_{old}$$

Iterative computation

$$\begin{bmatrix} n \\ m \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \\ 3/2 \end{bmatrix} \begin{bmatrix} 5/4 \\ 3/4 \\ 1 \end{bmatrix} \begin{bmatrix} 9/8 \\ 1/2 \\ 11/8 \end{bmatrix} \begin{bmatrix} 5/4 \\ 11/16 \\ 17/16 \end{bmatrix} \xrightarrow{\infty} \begin{bmatrix} 6/5 \\ 3/5 \\ 6/5 \end{bmatrix}$$

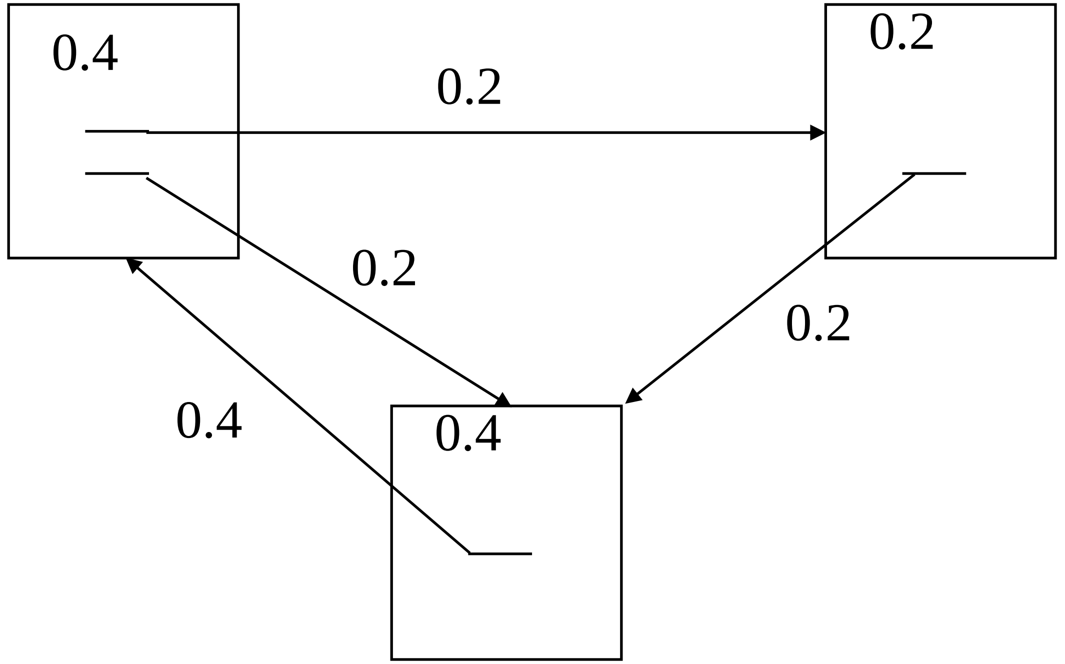


Final result:

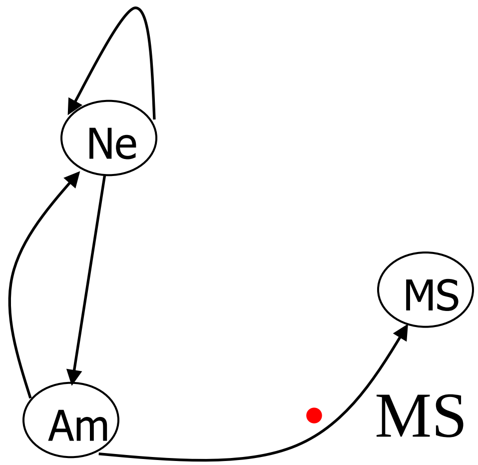
- Netscape and Amazon have the same importance, and twice the importance of Microsoft.

Another web graph with rank values

Converged results from iteration computation are marked



Problem 1 of algorithm: dead ends!

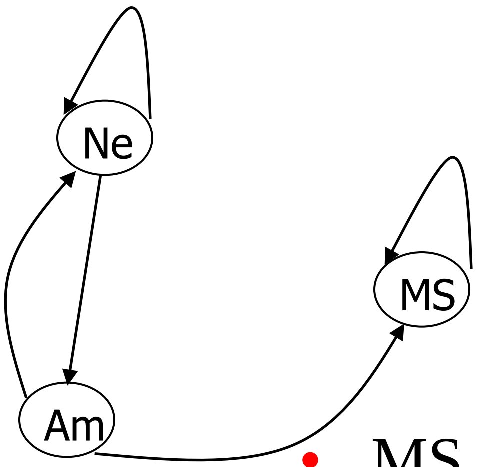


$$\begin{bmatrix} n \\ m \\ a \end{bmatrix}_{new} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 \\ 1/2 & 0 & 0 \end{pmatrix} \begin{bmatrix} n \\ m \\ a \end{bmatrix}_{old}$$

- MS does not point to anybody
- Result: weights of the Web “leak out”

$$\begin{bmatrix} n \\ m \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \\ 1/2 \end{bmatrix} \begin{bmatrix} 3/4 \\ 1/4 \\ 1/2 \end{bmatrix} \begin{bmatrix} 5/8 \\ 1/4 \\ 3/8 \end{bmatrix} \begin{bmatrix} 1/2 \\ 3/16 \\ 5/16 \end{bmatrix} \xrightarrow{\infty} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Problem 2 of algorithm: spider traps



$$\begin{bmatrix} n \\ m \\ a \end{bmatrix}_{new} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 1/2 & 0 & 0 \end{pmatrix} \begin{bmatrix} n \\ m \\ a \end{bmatrix}_{old}$$

- MS only points to itself
- Result: all weights go to MS!

$$\begin{bmatrix} n \\ m \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3/2 \\ 1/2 \end{bmatrix} \begin{bmatrix} 3/4 \\ 7/4 \\ 1/2 \end{bmatrix} \begin{bmatrix} 5/8 \\ 2 \\ 3/8 \end{bmatrix} \begin{bmatrix} 1/2 \\ 35/16 \\ 5/16 \end{bmatrix} \xrightarrow{\infty} \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

A revised solution with matrix notation: *r=Ar+c*

- Matrix *A*: web connectivity matrix
- Portion of each page's rank comes from a fixed weight *c*
  - Example: 0.2.

$$\begin{bmatrix} n \\ m \\ a \end{bmatrix} = 0.8 * \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 1/2 & 0 & 0 \end{pmatrix} \begin{bmatrix} n \\ m \\ a \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$$

- Rank result *r* from iterative computation converges to

$$\begin{bmatrix} n \\ m \\ a \end{bmatrix} = \begin{bmatrix} 7/11 \\ 21/11 \\ 5/11 \end{bmatrix}$$