Boolean and Vector Space Retrieval Models

- CS 290N
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Table of Content

Which results satisfy the query constraint?

- **Boolean model**
- **Statistical vector space model**

Retrieval Models

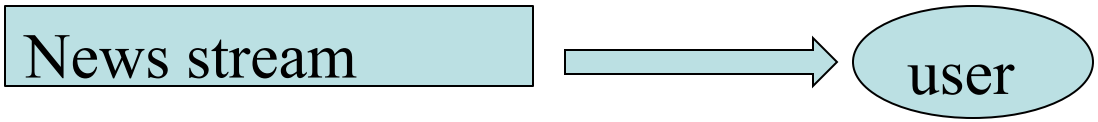
- **A retrieval model specifies the details of:**
  - Document representation
  - Query representation
  - Retrieval function: how to find relevant results
- **Determines a notion of relevance.**
  - Notion of relevance can be binary or continuous

Classes of Retrieval Models

- **Boolean models (set theoretic)**
  - Extended Boolean
- **Vector space models (statistical/algebraic)**
  - Generalized VS
  - Latent Semantic Indexing
- **Probabilistic models**

Retrieval Tasks

- **Ad hoc retrieval: Fixed document corpus, varied queries.**
- **Filtering: Fixed query, continuous document stream.**
  - User Profile: A model of relative static preferences.
  - Binary decision of relevant/not-relevant.



- **Routing: Same as filtering but continuously supply ranked lists rather than binary filtering.**

Common Document Preprocessing Steps

- Strip unwanted characters/markup (e.g. HTML tags, punctuation, numbers, etc.).
- Break into tokens (keywords) on whitespace.
- Possibly use stemming and remove common stopwords (e.g. a, the, it, etc.).
- Detect common phrases (possibly using a domain specific dictionary).
- Build inverted index (keyword → list of docs containing it).

Boolean Model

- **A document is represented as a set of keywords.**
- **Queries are Boolean expressions of keywords, connected by AND, OR, and NOT, including the use of brackets to indicate scope.**
  - [[Rio & Brazil] | [Hilo & Hawaii]] & hotel & !Hilton
- **Output: Document is relevant or not. No partial matches or ranking.**
- **Popular retrieval model because:**
  - Easy to understand for simple queries.
  - Clean formalism.
- Boolean models can be extended to include ranking.

Query example: Shakespeare plays

- **Which plays of Shakespeare contain the words *Brutus AND Caesar* but *NOT Calpurnia*?**
- **Could grep all of Shakespeare's plays for *Brutus* and *Caesar,* then strip out lines containing *Calpurnia*?**
  - Slow (for large corpora)
  - <u>*NOT*</u> ***Calpurnia*** is non-trivial
  - Other operations (e.g., find the phrase ***Romans and countrymen***) not feasible

Term-document incidence

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

1 if play contains word, 0 otherwise

Incidence vectors

- **So we have a 0/1 vector for each term.**
- **To answer query: take the vectors for *Brutus, Caesar* and *Calpurnia* (complemented) → bitwise *AND*.**
- **110100 *AND* 110111 *AND* 101111 = 100100.**

Inverted index

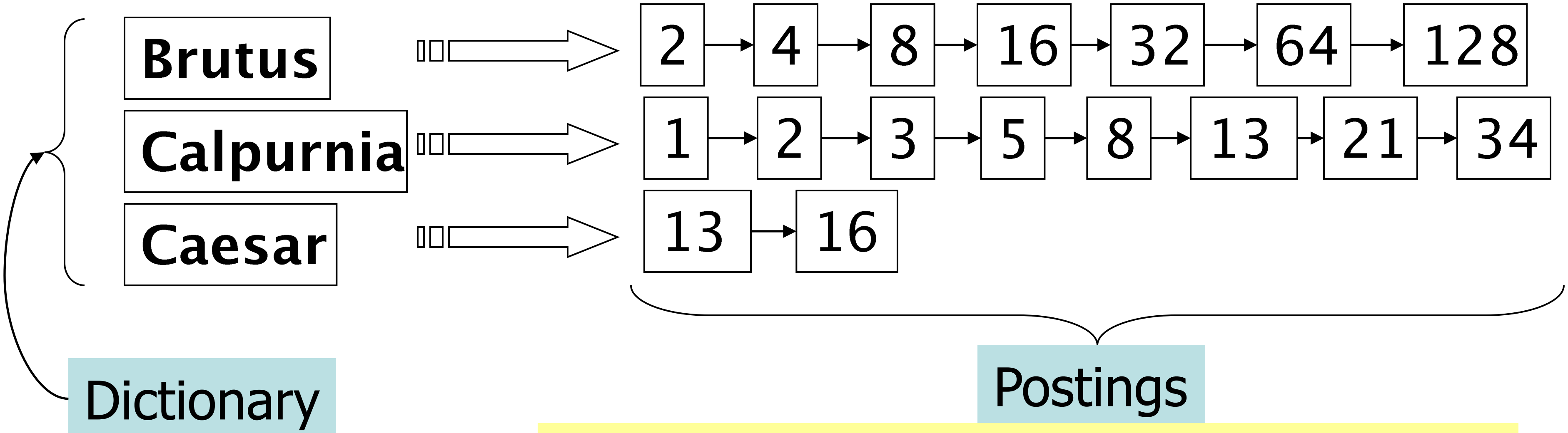
- **For each term *T*, must store a list of all documents that contain *T*.**

Brutus	2	4	8	16	32	64	128	
Calpurnia	1	2	3	5	8	13	21	34
Caesar	13	16						

What happens if the word **Caesar** is added to document 14?

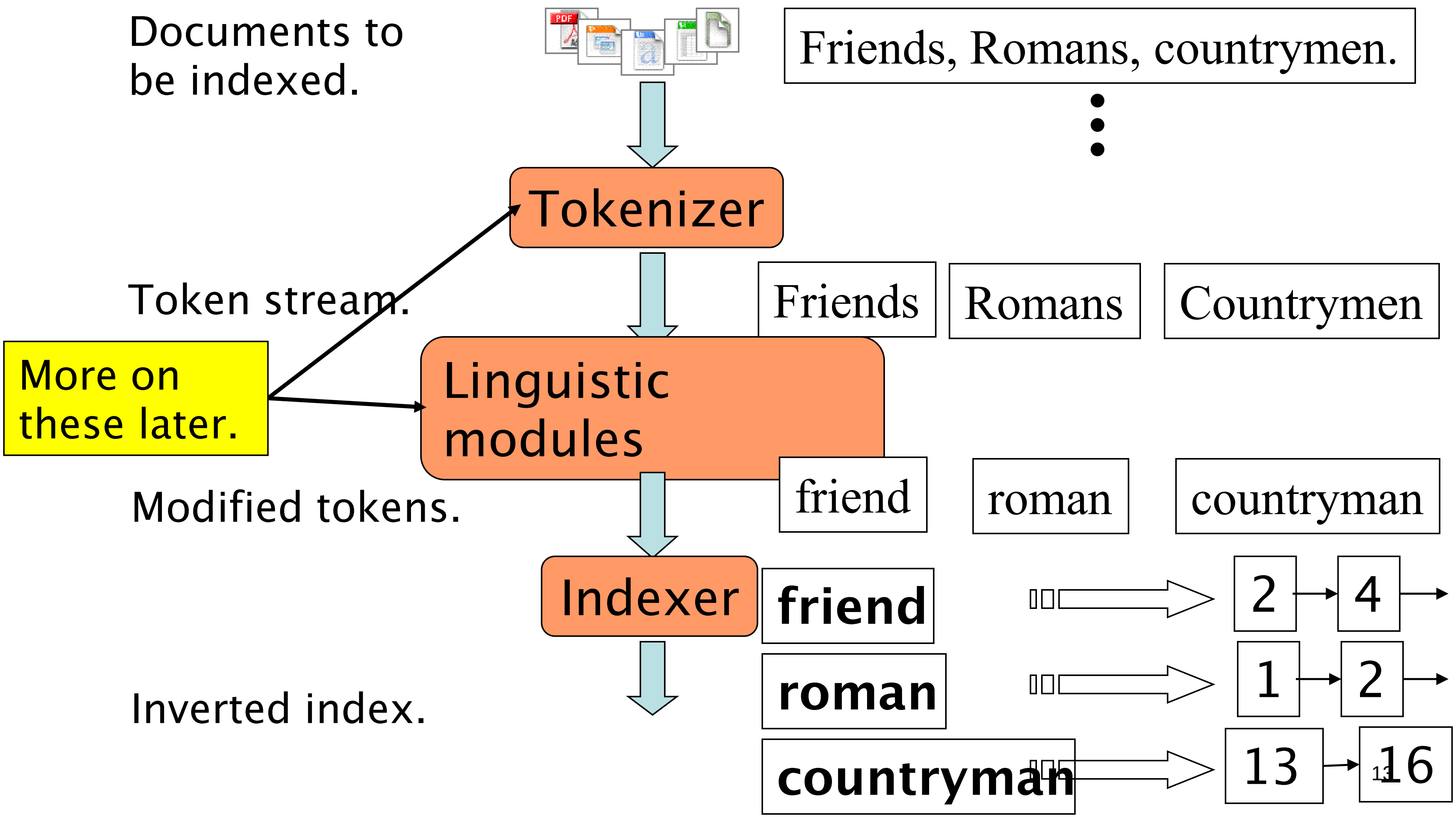
Inverted index

- **Linked lists generally preferred to arrays**
  - Dynamic space allocation
  - Insertion of terms into documents easy
  - Space overhead of pointers



Sorted by docID (more later on why).

Inverted index construction



Discussions

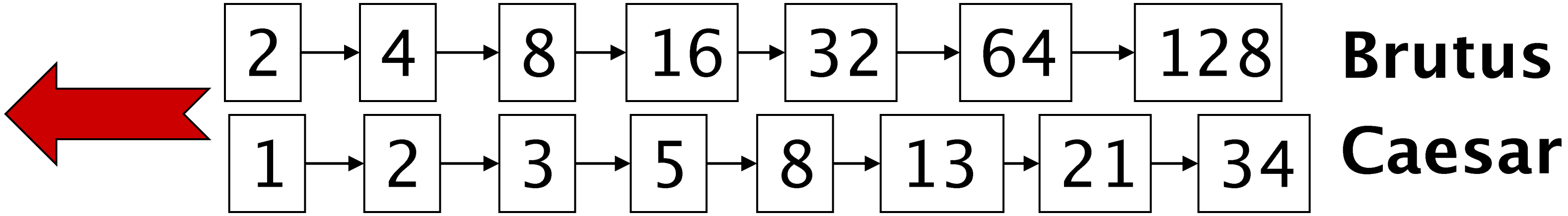
- **Which terms in a doc do we index?**
  - All words or only “important” ones?
- **<u>Stopword</u> list: terms that are so common**
  - they MAY BE ignored for indexing.
  - *e.g.*, ***the, a, an, of, to*** …
  - language-specific.
  - May have to be included for general web search
- **How do we process a query?**
  - What kinds of queries can we process?

Query processing

- **Consider processing the query:**

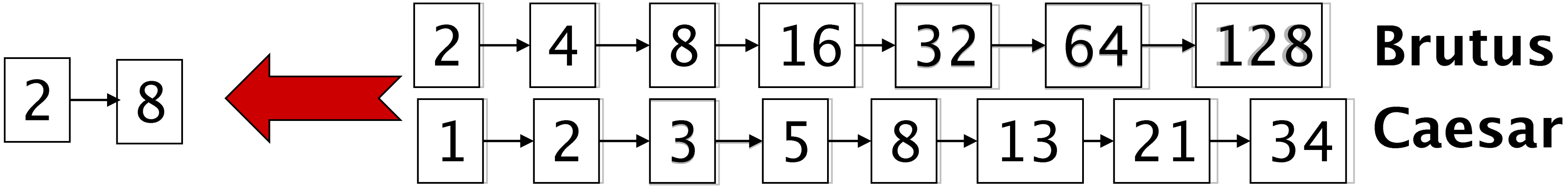
  Brutus *AND* ***Caesar***

  - Locate ***Brutus*** in the Dictionary;
    - Retrieve its postings.
  - Locate *Caesar* in the Dictionary;
    - Retrieve its postings.
  - "Merge" the two postings:



The merge

- **Walk through the two postings simultaneously, in time linear in the total number of postings entries**



If the list lengths are m and n, the merge takes O(m+n) operations.

Crucial: postings sorted by docID.

Example: WestLaw http://www.westlaw.com/

- **Largest commercial (paying subscribers) legal search service (started 1975; ranking added 1992)**
- **Majority of users *still* use boolean queries**
- **Example query:**
  - What is the statute of limitations in cases involving the federal tort claims act?
  - LIMIT! /3 STATUTE ACTION /S FEDERAL /2 TORT /3 CLAIM
- **Long, precise queries; proximity operators; incrementally developed; not like web search**
  - Professional searchers (e.g., Lawyers) still like Boolean queries:
  - You know exactly what you're getting.

More general merges

- **Exercise: Adapt the merge for the queries:**

  Brutus AND NOT Caesar

  Brutus OR NOT Caesar

Can we still run through the merge in time O(*m*+*n*)?

Boolean Models – Problems

- **Very rigid: AND means all; OR means any.**
- **Difficult to express complex user requests.**
- **Difficult to control the number of documents retrieved.**
  - *All* matched documents will be returned.
- **Difficult to rank output.**
  - *All* matched documents logically satisfy the query.
- **Difficult to perform relevance feedback.**
  - If a document is identified by the user as relevant or irrelevant, how should the query be modified?

Statistical Retrieval Models

- **A document is typically represented by a *bag of words* (unordered words with frequencies).**
- **Bag = set that allows multiple occurrences of the same element.**
- **User specifies a set of desired terms with optional weights:**
  - Weighted query terms:

    Q = < database 0.5; text 0.8; information 0.2 >
  - Unweighted query terms:

    Q = < database; text; information >
  - No Boolean conditions specified in the query.

Statistical Retrieval

- **Retrieval based on *similarity* between query and documents.**
- **Output documents are ranked according to similarity to query.**
- **Similarity based on occurrence *frequencies* of keywords in query and document.**
- **Automatic relevance feedback can be supported:**
  - Relevant documents “added” to query.
  - Irrelevant documents “subtracted” from query.

The Vector-Space Model

- **Assume *t* distinct terms remain after preprocessing; call them index terms or the vocabulary.**
- **These "orthogonal" terms form a vector space.**

  Dimension = t = |vocabulary|

- **Each term, *i*, in a document or query, *j*, is given a real-valued weight, w_{ij}.**
- **Both documents and queries are expressed as t-dimensional vectors:**

  $d_j = (w_{1j}, w_{2j}, \ldots, w_{tj})$

Document Collection

- A collection of *n* documents can be represented in the vector space model by a term-document matrix.
- An entry in the matrix corresponds to the "weight" of a term in the document; zero means the term has no significance in the document or it simply doesn't exist in the document.

$$
\begin{array}{c|cccc}
 & T_1 & T_2 & \dots & T_t \\
\hline
D_1 & w_{11} & w_{21} & \dots & w_{t1} \\
D_2 & w_{12} & w_{22} & \dots & w_{t2} \\
: & : & : & & : \\
: & : & : & & : \\
D_n & w_{1n} & w_{2n} & \dots & w_{tn}
\end{array}
$$

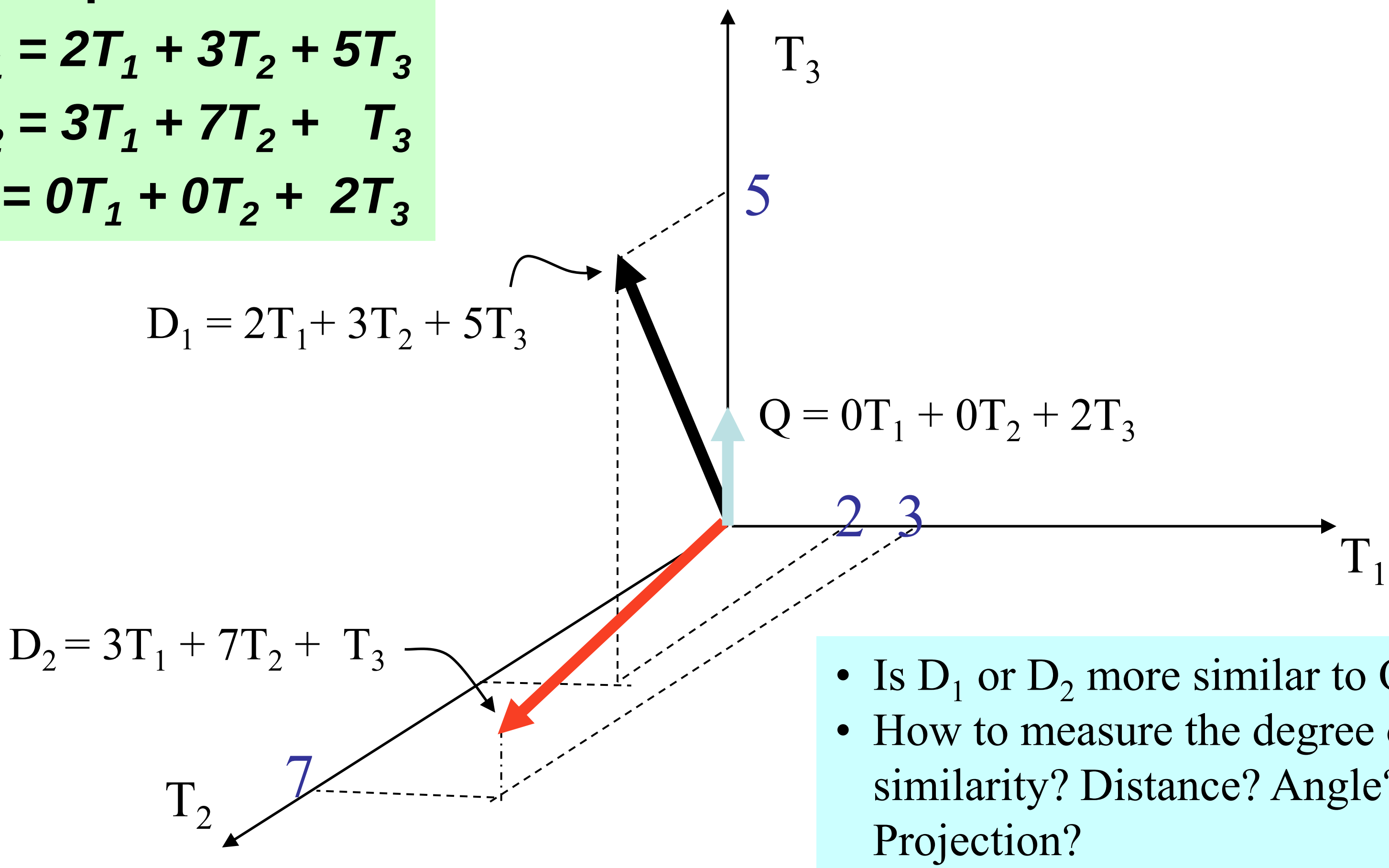
Graphic Representation

Example*:*

$D_1 = 2T_1 + 3T_2 + 5T_3$

$D_2 = 3T_1 + 7T_2 + T_3$

$Q = 0T_1 + 0T_2 + 2T_3$



- Is D_1 or D_2 more similar to Q?
- How to measure the degree of similarity? Distance? Angle? Projection?

Issues for Vector Space Model

- **How to determine important words in a document?**
  - Word n-grams (and phrases, idioms,…) → terms
- **How to determine the degree of importance of a term within a document and within the entire collection?**
- **How to determine the degree of similarity between a document and the query?**
- **In the case of the web, what is a collection and what are the effects of links, formatting information, etc.?**

Term Weights: Term Frequency

- **More frequent terms in a document are more important, i.e. more indicative of the topic.**

$$f_{ij} = \text{frequency of term } i \text{ in document } j$$

- **May want to normalize *term frequency* (*tf*) across the entire corpus:**

$$tf_{ij} = f_{ij} \; / \; max\{f_{ij}\}$$

Term Weights: Inverse Document Frequency

- **Terms that appear in many *different* documents are *less* indicative of overall topic.**

  df_i **= document frequency of term i**

  = number of documents containing term i

  idf_i **= inverse document frequency of term i,**

  $= \log_2 (N / df_i)$

  (N: total number of documents)

- **An indication of a term's *discrimination* power.**
- **Log used to dampen the effect relative to *tf*.**

TF-IDF Weighting

- A typical combined term importance indicator is ***tf-idf weighting***:

$$w_{ij} = tf_{ij}\, idf_i = tf_{ij} \log_2 (N/ df_i)$$

- A term occurring frequently in the document but rarely in the rest of the collection is given high weight.
- Many other ways of determining term weights have been proposed.
- Experimentally, *tf-idf* has been found to work well.

Computing TF-IDF -- An Example

Given a document with term frequencies:

A(3), B(2), C(1)

Assume collection contains 10,000 documents and document frequencies of these terms are:

A(50), B(1300), C(250)

Then:

A: tf = 3/3; idf = log(10000/50) = 5.3; tf-idf = 5.3

B: tf = 2/3; idf = log(10000/1300) = 2.0; tf-idf = 1.3

C: tf = 1/3; idf = log(10000/250) = 3.7; tf-idf = 1.2

Similarity Measure

- **A similarity measure is a function that computes the *degree of similarity* between two vectors.**

- **Using a similarity measure between the query and each document:**
  - It is possible to rank the retrieved documents in the order of presumed relevance.
  - It is possible to enforce a certain threshold so that the size of the retrieved set can be controlled.

Similarity Measure - Inner Product

- **Similarity between vectors for the document d_i and query q can be computed as the vector inner product:**

$$\mathbf{sim}(d_j,q) = d_j \bullet q = \sum_{i=1}^{t} w_{ij} \cdot w_{iq}$$

where w_{ij} is the weight of term i in document j and w_{iq} is the weight of term i in the query

- **For binary vectors, the inner product is the number of matched query terms in the document (size of intersection).**
- **For weighted term vectors, it is the sum of the products of the weights of the matched terms.**

Properties of Inner Product

- **The inner product is unbounded.**

- **Favors long documents with a large number of unique terms.**

- **Measures how many terms matched but not how many terms are *not* matched.**

Inner Product -- Examples

Binary:

database retrieval architecture computer information management text

- D = 1, 1, 1, 0, 1, 1, 0
- Q = 1, 0 , 1, 0, 0, 1, 1

sim(D, Q) = 3

Weighted:

$D_1 = 2T_1 + 3T_2 + 5T_3$ $\quad$ $D_2 = 3T_1 + 7T_2 + 1T_3$

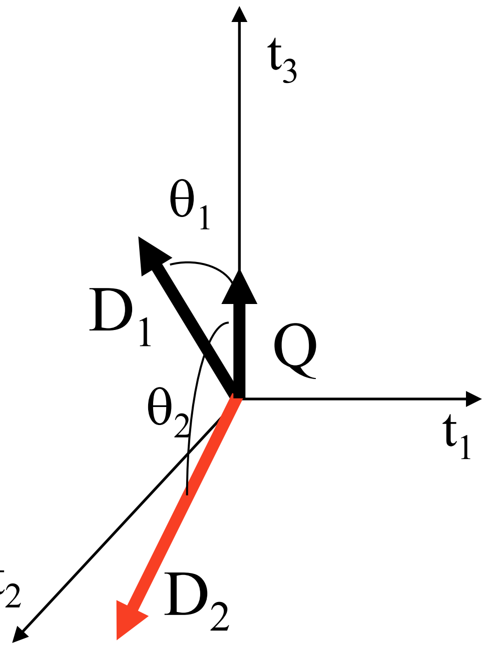
$Q = 0T_1 + 0T_2 + 2T_3$

$\mathrm{sim}(D_1, Q) = 2*0 + 3*0 + 5*2 = 10$

$\mathrm{sim}(D_2, Q) = 3*0 + 7*0 + 1*2 = 2$

Cosine Similarity Measure

- **Cosine similarity measures the cosine of the angle between two vectors.**
- **Inner product normalized by the vector lengths.**



$$\text{CosSim}(\mathbf{d}_j, \mathbf{q}) = \frac{\vec{d_j} \cdot \vec{q}}{\left|\vec{d_j}\right| \cdot \left|\vec{q}\right|} = \frac{\sum_{i=1}^{t} (w_{ij} \cdot w_{iq})}{\sqrt{\sum_{i=1}^{t} w_{ij}^2 \cdot \sum_{i=1}^{t} w_{iq}^2}}$$

$D_1 = 2T_1 + 3T_2 + 5T_3$ CosSim(D_1 , Q) = 10 / $\sqrt{(4+9+25)(0+0+4)}$ = 0.81
$D_2 = 3T_1 + 7T_2 + 1T_3$ CosSim(D_2 , Q) = 2 / $\sqrt{(9+49+1)(0+0+4)}$ = 0.13
$Q = 0T_1 + 0T_2 + 2T_3$

D_1 is 6 times better than D_2 using cosine similarity but only 5 times better using inner product.

Comments on Vector Space Models

- Simple, mathematically based approach.
- Considers both local (*tf*) and global (*idf*) word occurrence frequencies.
- Provides partial matching and ranked results.
- Tends to work quite well in practice despite obvious weaknesses.
- Allows efficient implementation for large document collections.

Problems with Vector Space Model

- **Missing semantic information (e.g. word sense).**
- **Missing syntactic information (e.g. phrase structure, word order, proximity information).**
- **Assumption of term independence (e.g. ignores synonomy).**
- **Lacks the control of a Boolean model (e.g., *requiring* a term to appear in a document).**
  - Given a two-term query “A B”, may prefer a document containing A frequently but not B, over a document that contains both A and B, but both less frequently.