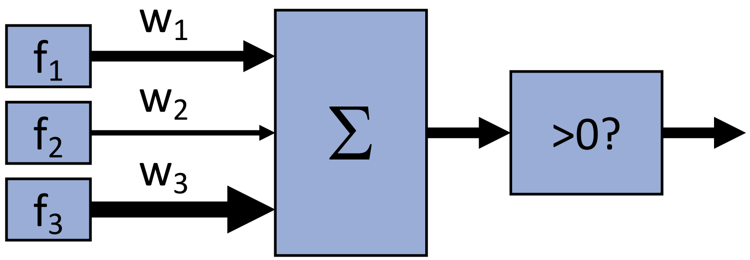
Deep Learning for Classification

CS293S, Yang, 2017

Computational graph for classification



- Objective: Classification Accuracy

$$l^{\text{acc}}(w) = \frac{1}{m}\sum_{i=1}^{m}\left(\text{sign}(w^\top f(x^{(i)})) == y^{(i)}\right)$$

  – Issue: How to find these parameteres?

Neural Net with Soft-Max

- Score for y=1: $w^\top f(x)$ Score for y=-1: $-w^\top f(x)$
- Probability of label:

$$p(y=1|f(x);w) = \frac{e^{w^\top f(x^{(i)})}}{e^{w^\top f(x)} + e^{-w^\top f(x)}}$$

$$p(y=-1|f(x);w) = \frac{e^{-w^\top f(x)}}{e^{w^\top f(x)} + e^{-w^\top f(x)}}$$

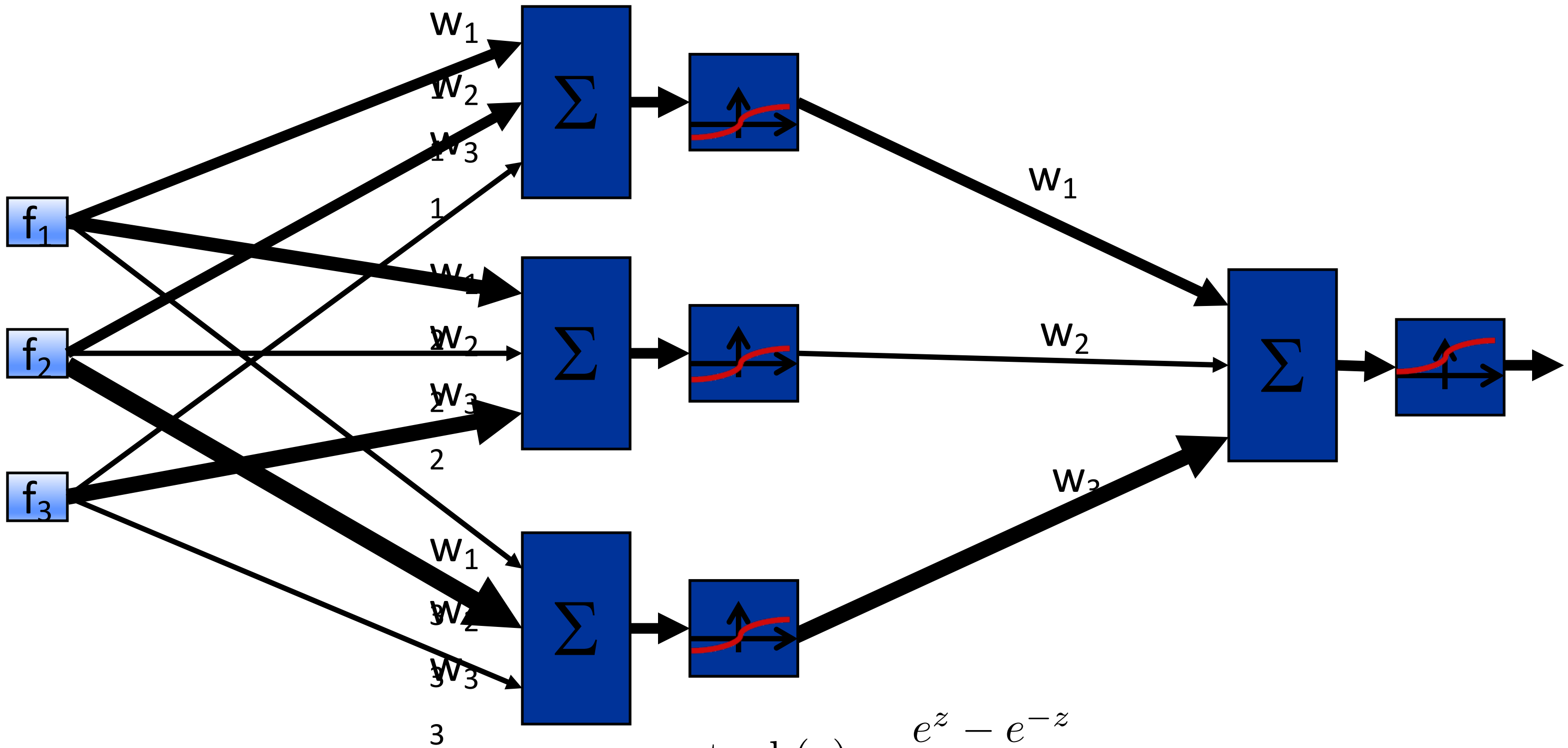
- Objective:

$$l(w) = \prod_{i=1}^{m} p(y = y^{(i)}|f(x^{(i)});w)$$

- Log:

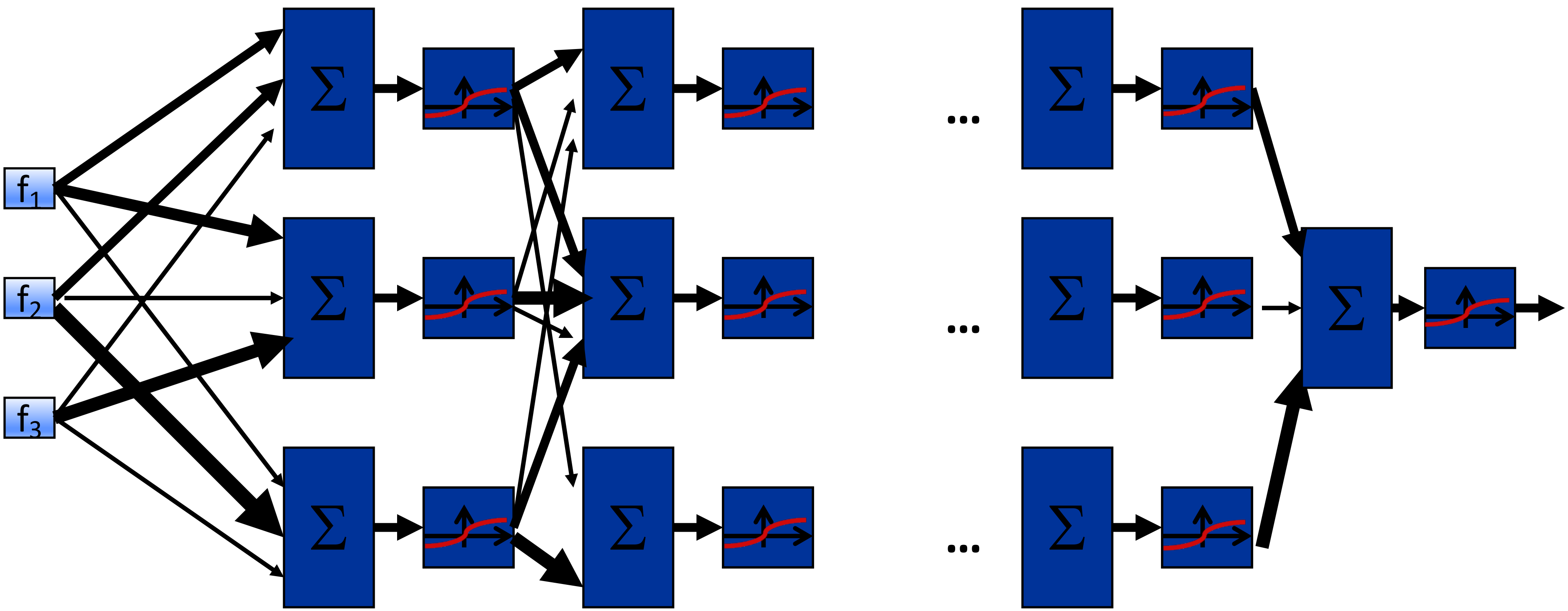
$$ll(w) = \sum_{i=1}^{m} \log p(y = y^{(i)}|f(x^{(i)});w)$$

Two-Layer Neural Network

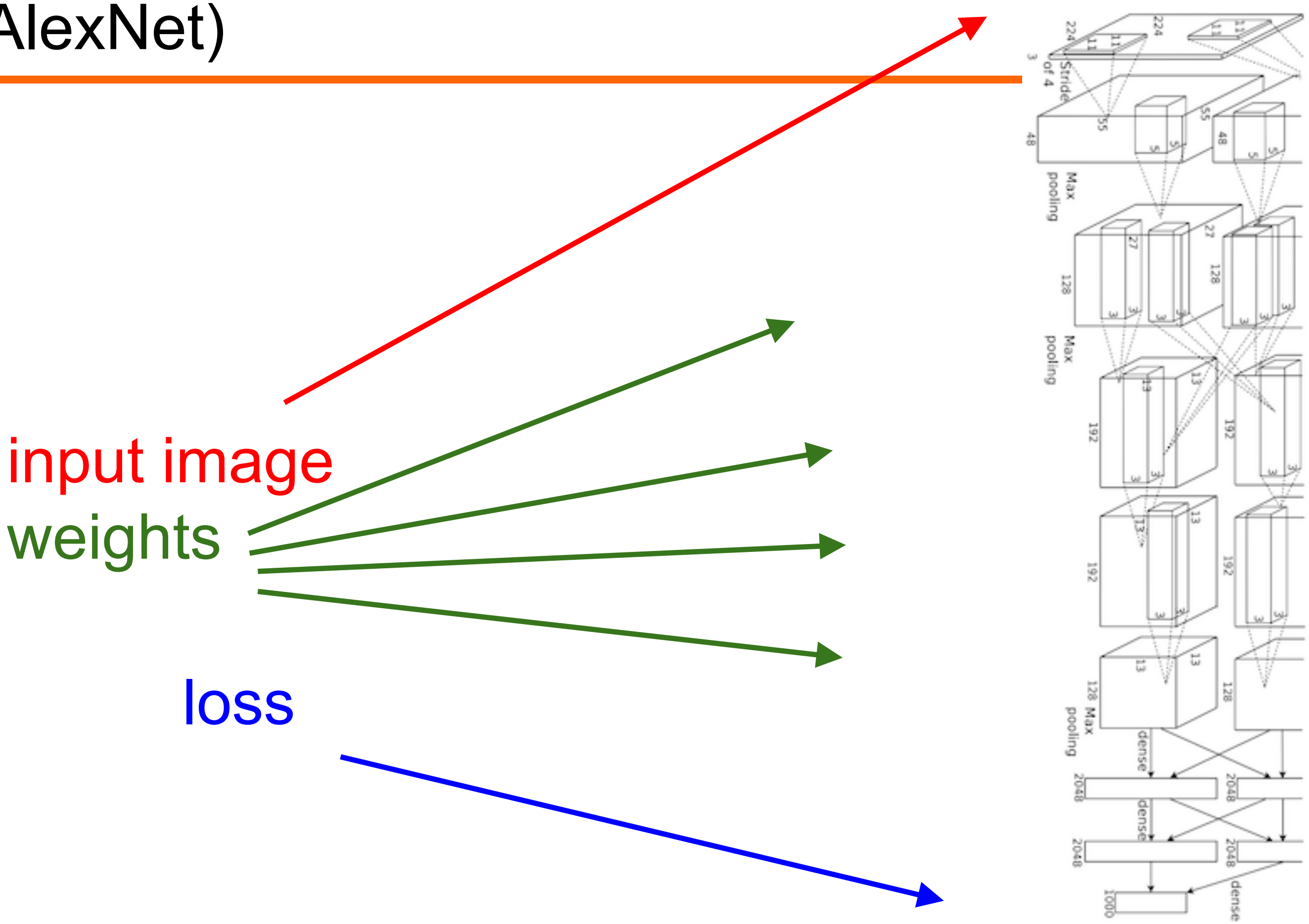


$$z \rightarrow \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

N-Layer Neural Network



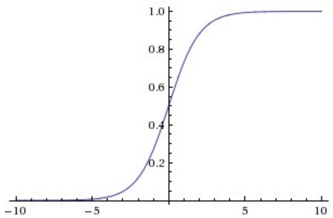
Convolutional Network (AlexNet)



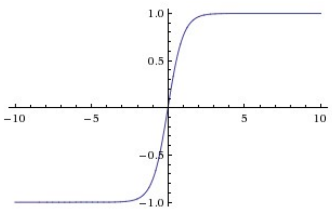
Activation Functions

Sigmoid

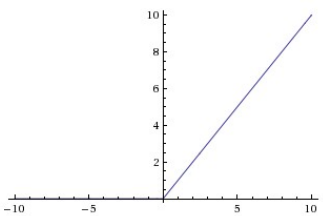
$$\sigma(x) = 1/(1+e^{-x})$$



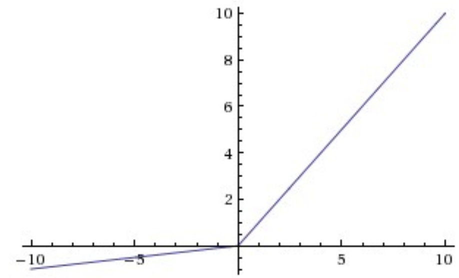
tanh tanh(x)



ReLU max(0,x)



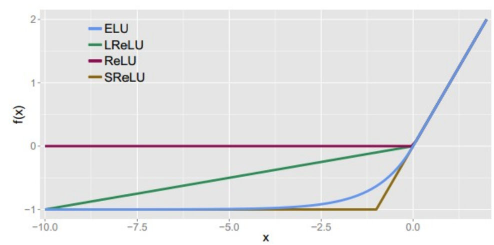
Leaky ReLU
max(0.1x, x)



Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$



Multi-class Softmax

- 3-class softmax – classes A, B, C
  - 3 weight vectors:

$$w_A, w_B, w_C$$

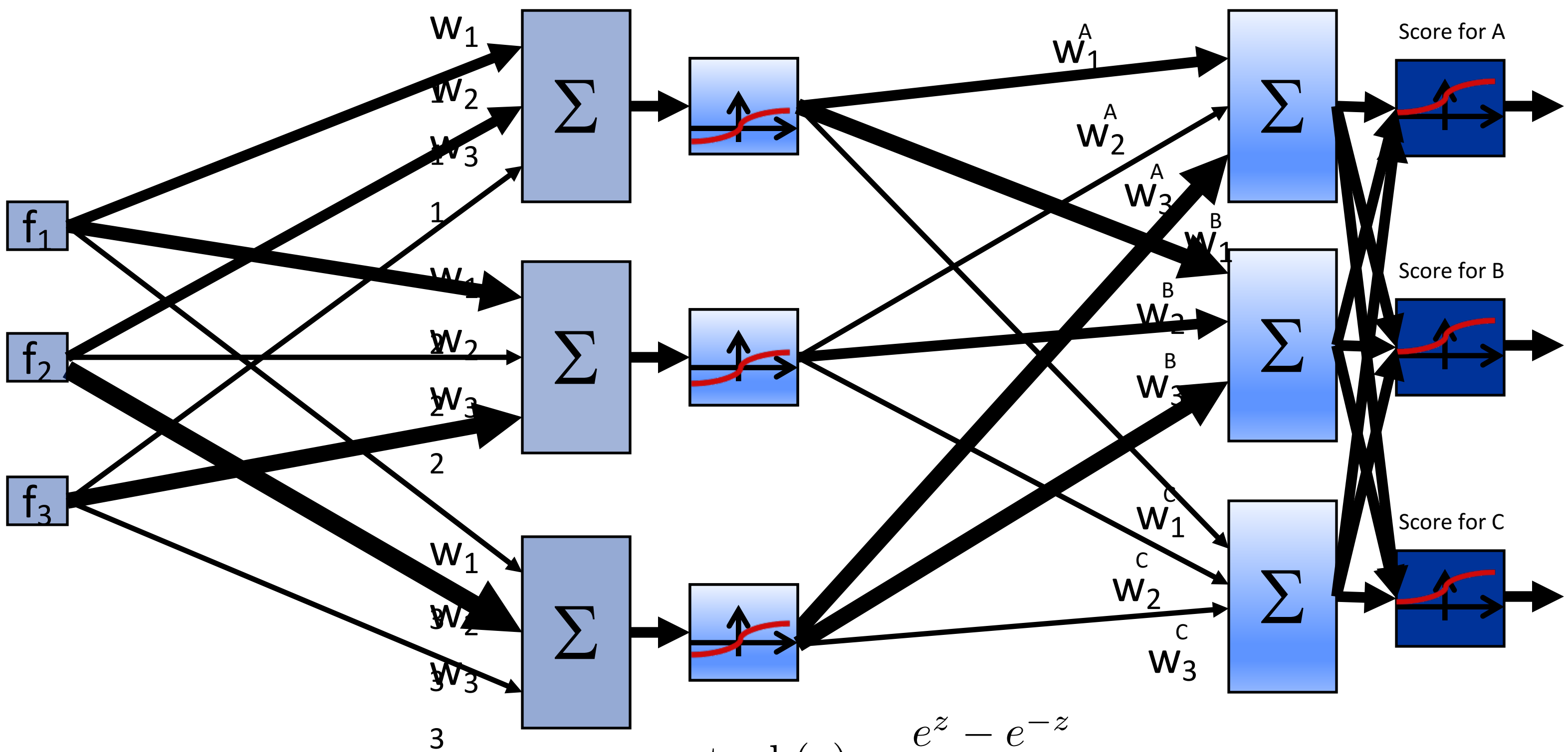
- Probability of label A: (similar for B, C)

$$p(y = A|f(x); w) = \frac{e^{w_A^\top f(x)}}{e^{w_A^\top f(x)} + e^{w_B^\top f(x)} + e^{w_C^\top f(x)}}$$

- Objective: $l(w) = \prod_{i=1}^{m} p(y = y^{(i)}|f(x^{(i)}; w)$

- Log: $$ll(w) = \sum_{i=1}^{m} \log p(y = y^{(i)}|f(x^{(i)}; w)$$

Multi-class Two-Layer Neural Network



$$z \rightarrow \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Gradient Descent Method for Optimization

- How to find parameters that minimize an objective function?
- Idea:
  - Start somewhere
  - Repeat: Take a step in the steepest descent direction

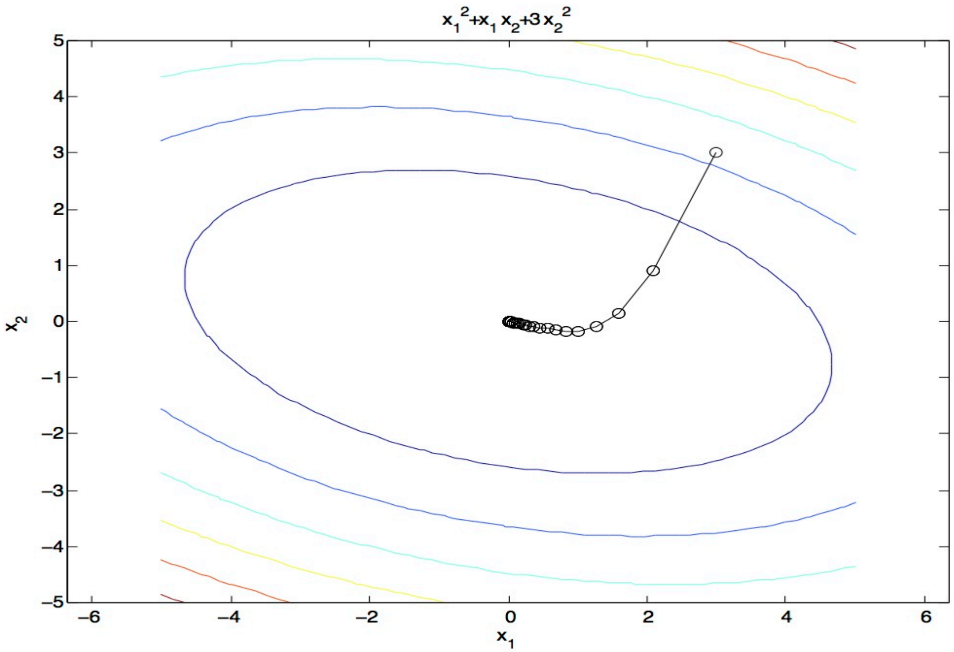


Figure source: Mathworks

Generally, Steepest Direction

- Steepest Direction = direction of the gradient

$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial w_1} \\ \frac{\partial g}{\partial w_2} \\ \cdots \\ \frac{\partial g}{\partial w_n} \end{bmatrix}$$

- Gradient Descent

- Init: w
- For i = 1, 2, …

$$w \leftarrow w - \alpha * \nabla g(w)$$

What is the Steepest Descent Direction?

$$\min_{\Delta:\Delta_1^2+\Delta_2^2\leq\epsilon} g(w+\Delta)$$

- First-Order Taylor Expansion: $g(w+\Delta) \approx g(w) + \frac{\partial g}{\partial w_1}\Delta_1 + \frac{\partial g}{\partial w_2}\Delta_2$
- Steepest Descent Direction: $\min_{\Delta:\Delta_1^2+\Delta_2^2\leq\epsilon} \frac{\partial g}{\partial w_1}\Delta_1 + \frac{\partial g}{\partial w_2}\Delta_2$
- Recall: $\min_{a:\|a\|\leq\epsilon} a^\top b$ → $a = -b\frac{\epsilon}{\|b\|}$
- Hence, solution: $-\nabla g \frac{\epsilon}{\|\nabla g\|}$ $\quad \nabla g = \begin{bmatrix} \frac{\partial g}{\partial w_1} \\ \frac{\partial g}{\partial w_2} \end{bmatrix}$

How to Calculate a Partial Deriviate in a Computational Graph

Given a function $f(x,y,z)= (x+y)z$,
What is the partial derivie of f with respect to x, y, z?

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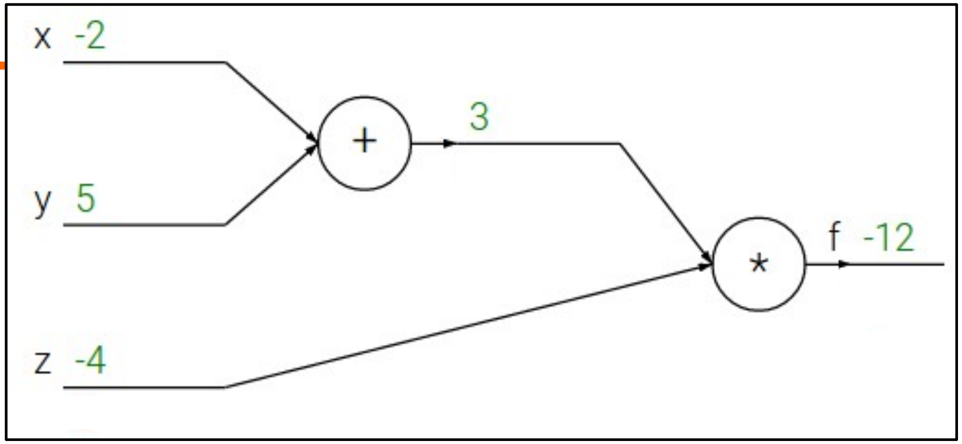
$$f(x,y,z) = (x+y)z$$

e.g. x = -2, y = 5, z = -4



x, y, z values are from a training example

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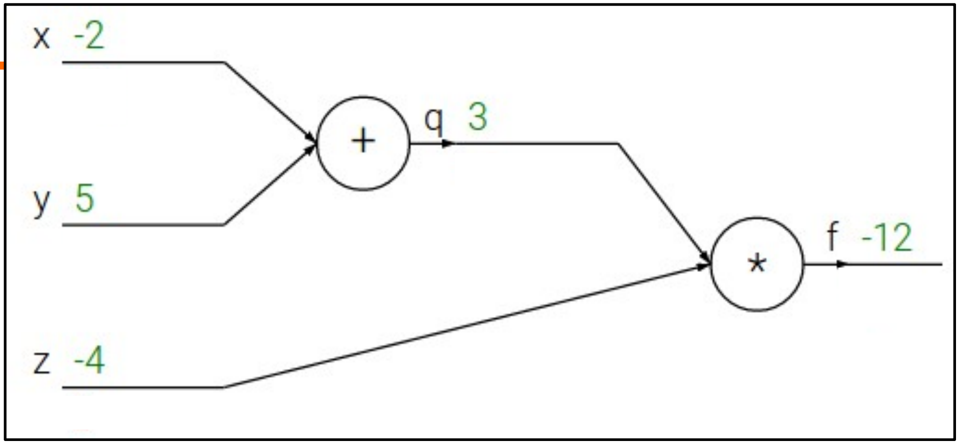

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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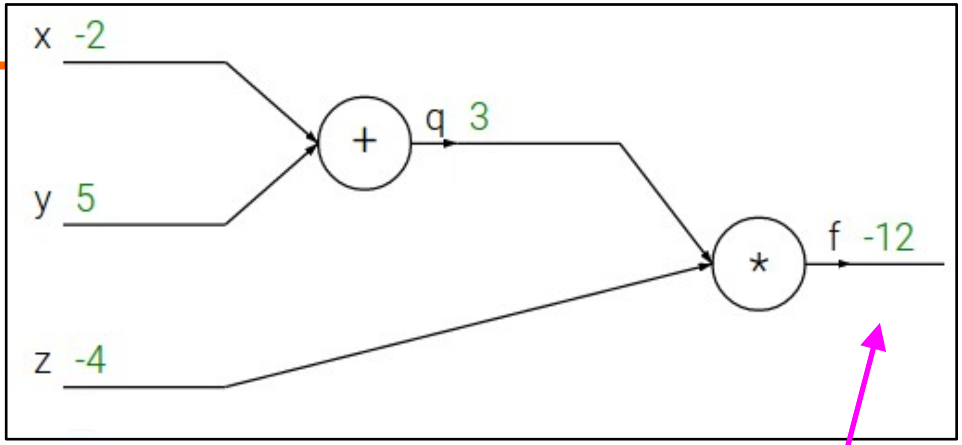

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$$\frac{\partial f}{\partial f}$$

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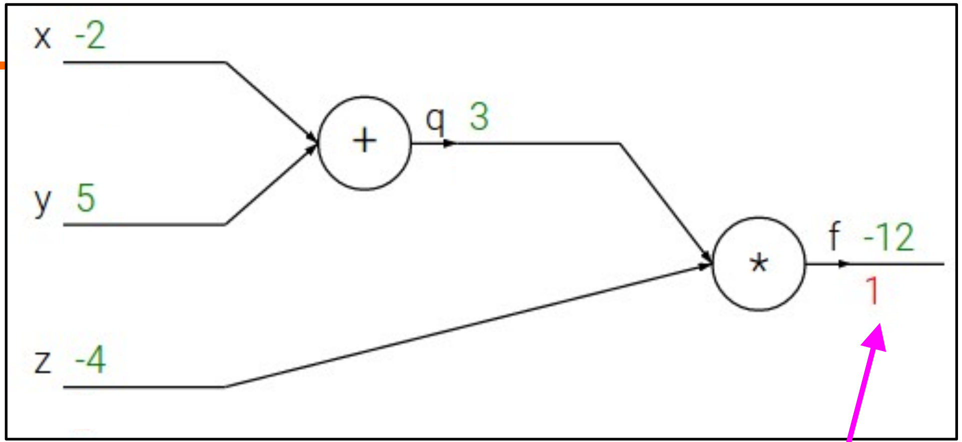

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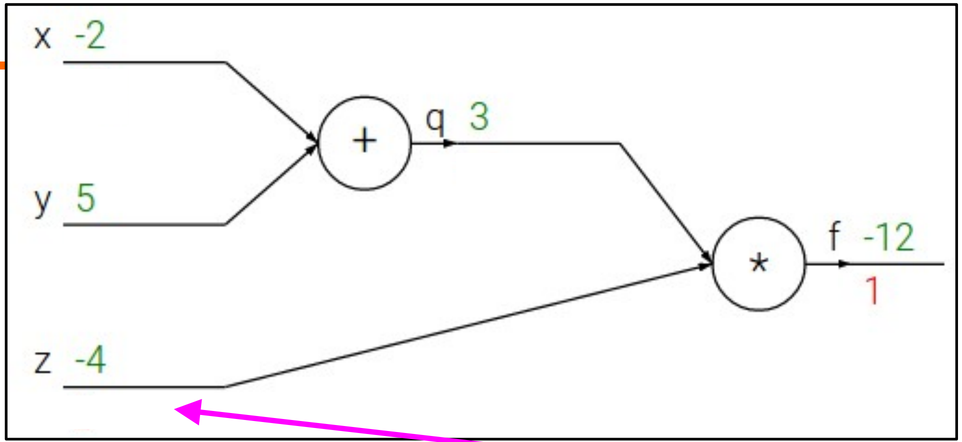

$$\frac{\partial f}{\partial z}$$

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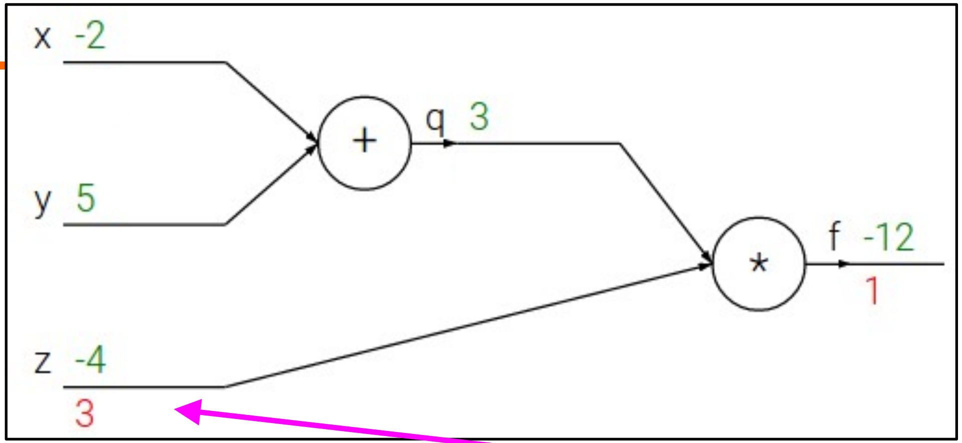

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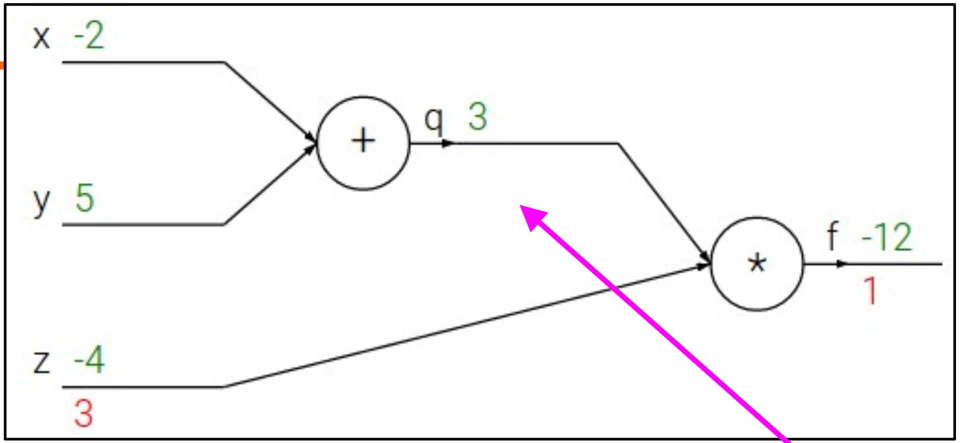

$$\frac{\partial f}{\partial q}$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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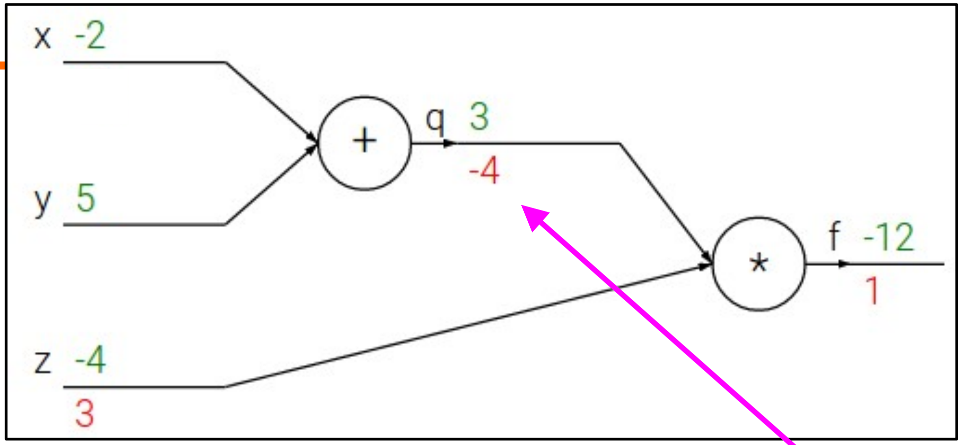

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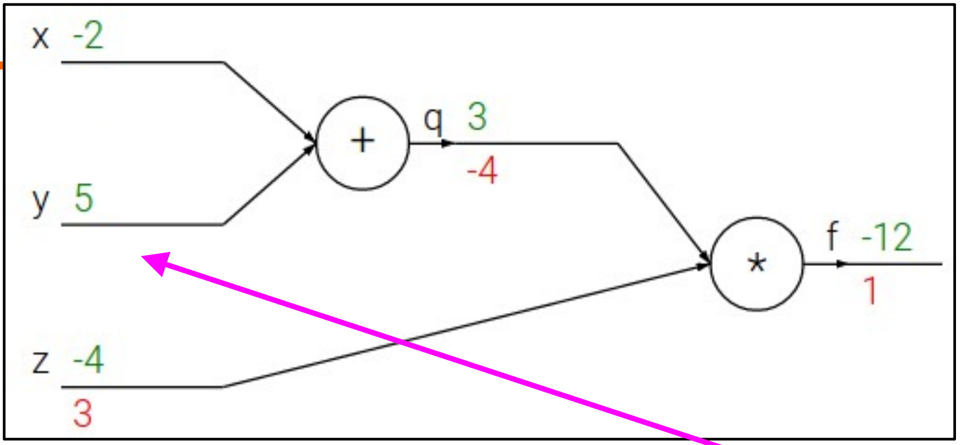

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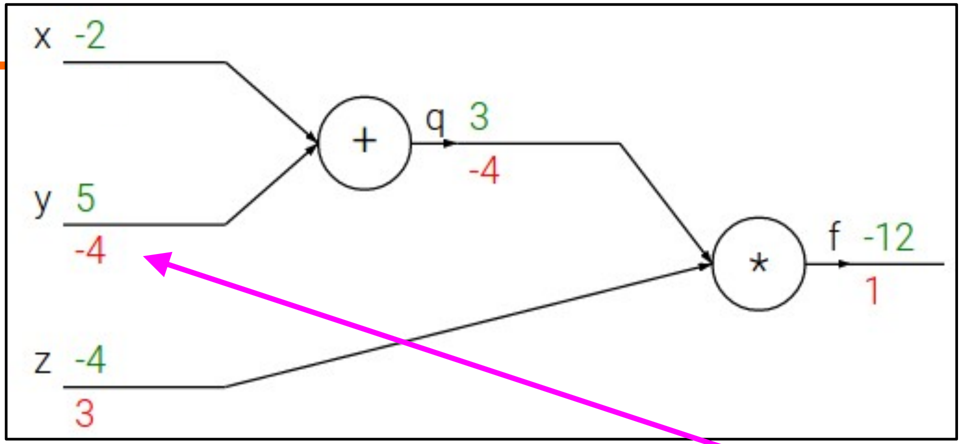

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$$\frac{\partial f}{\partial y}$$

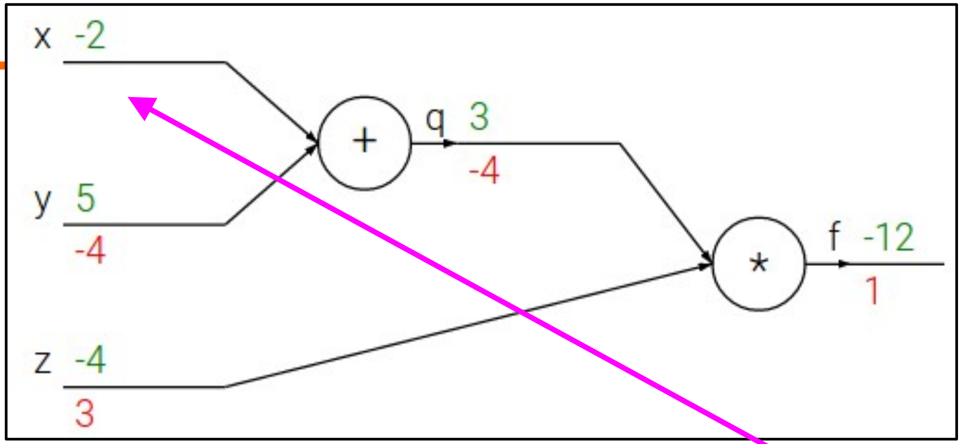
Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



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$$\frac{\partial f}{\partial x}$$

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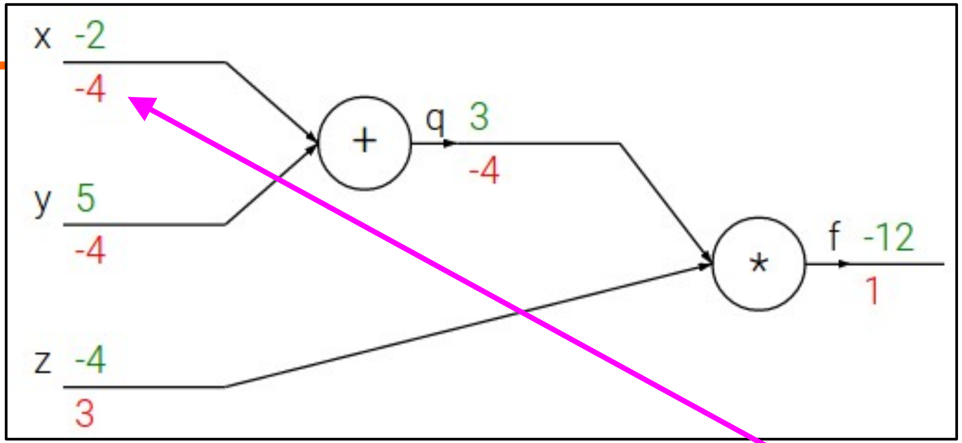

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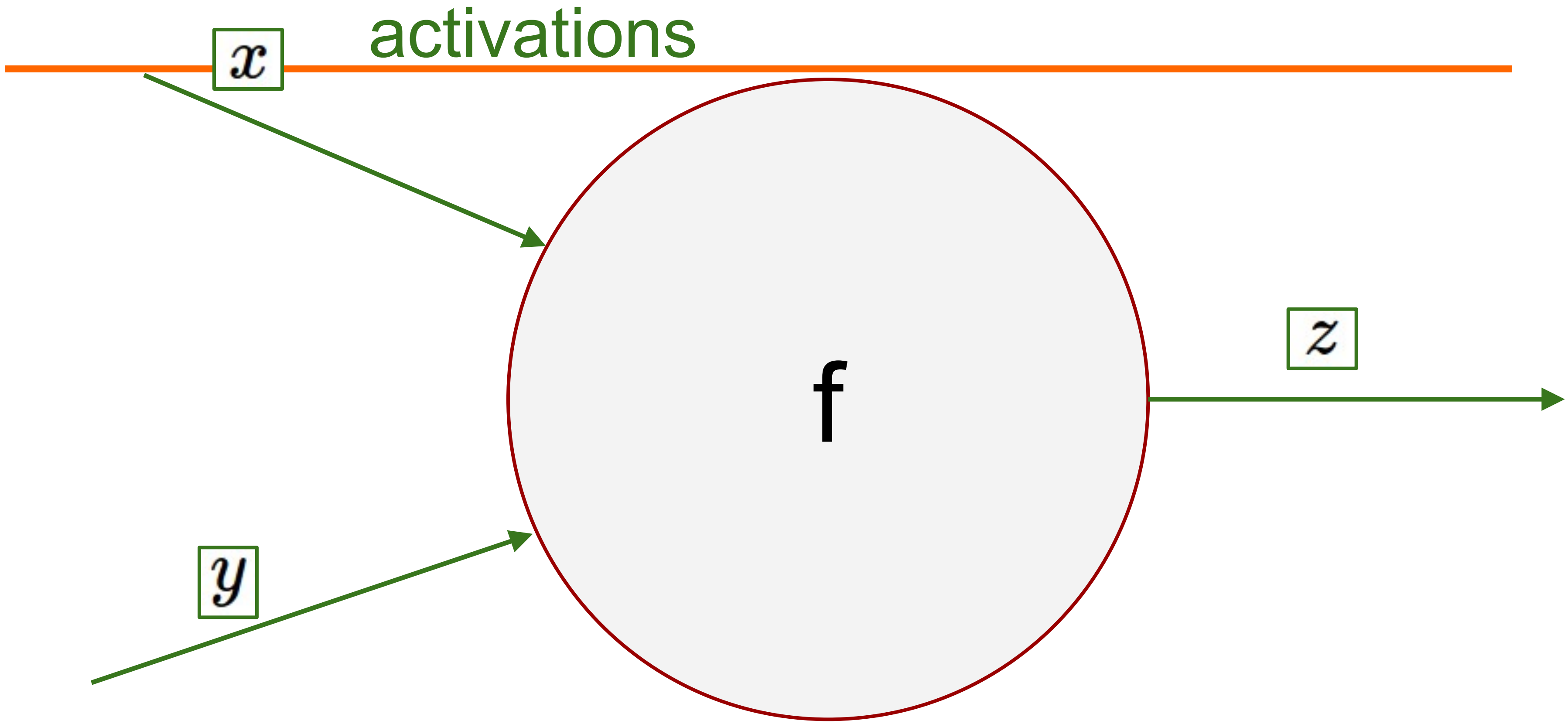
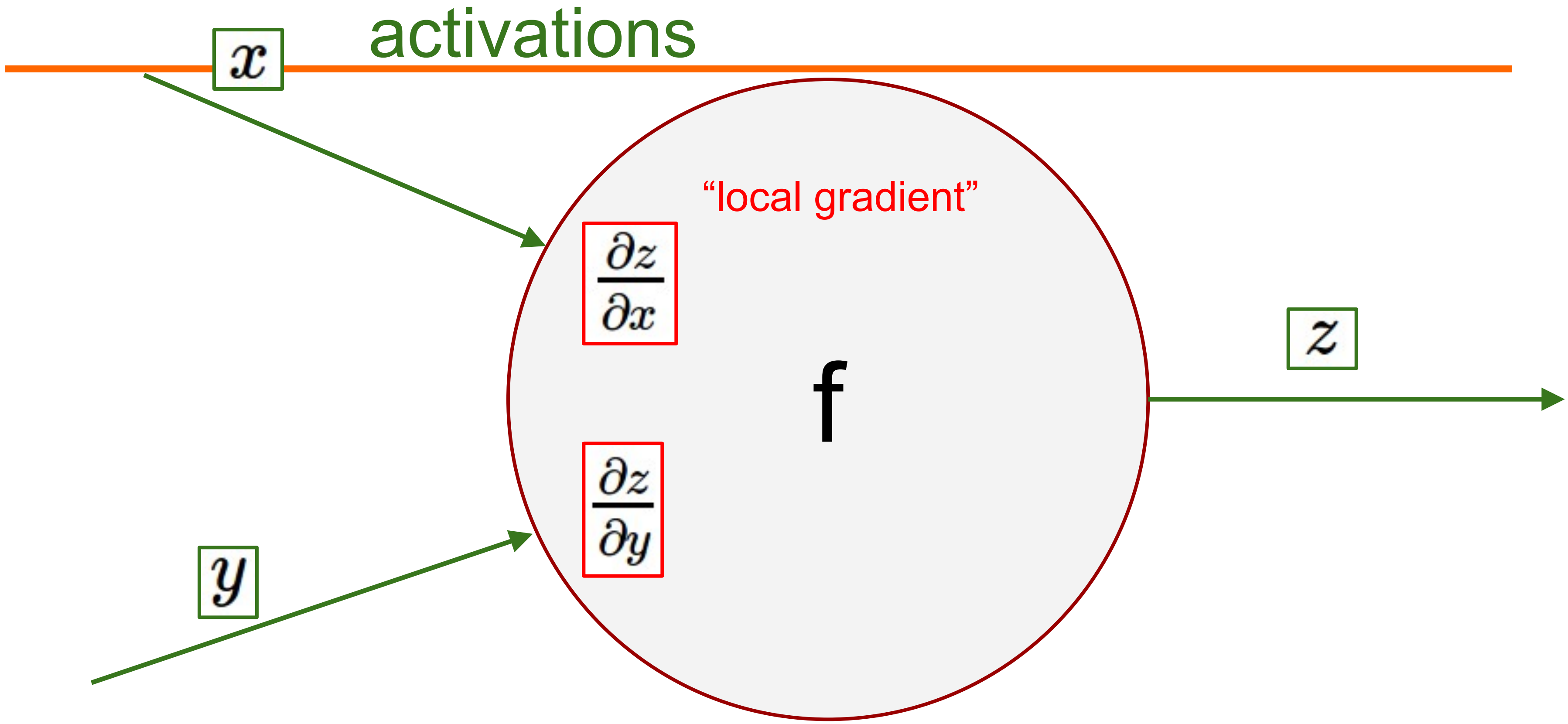
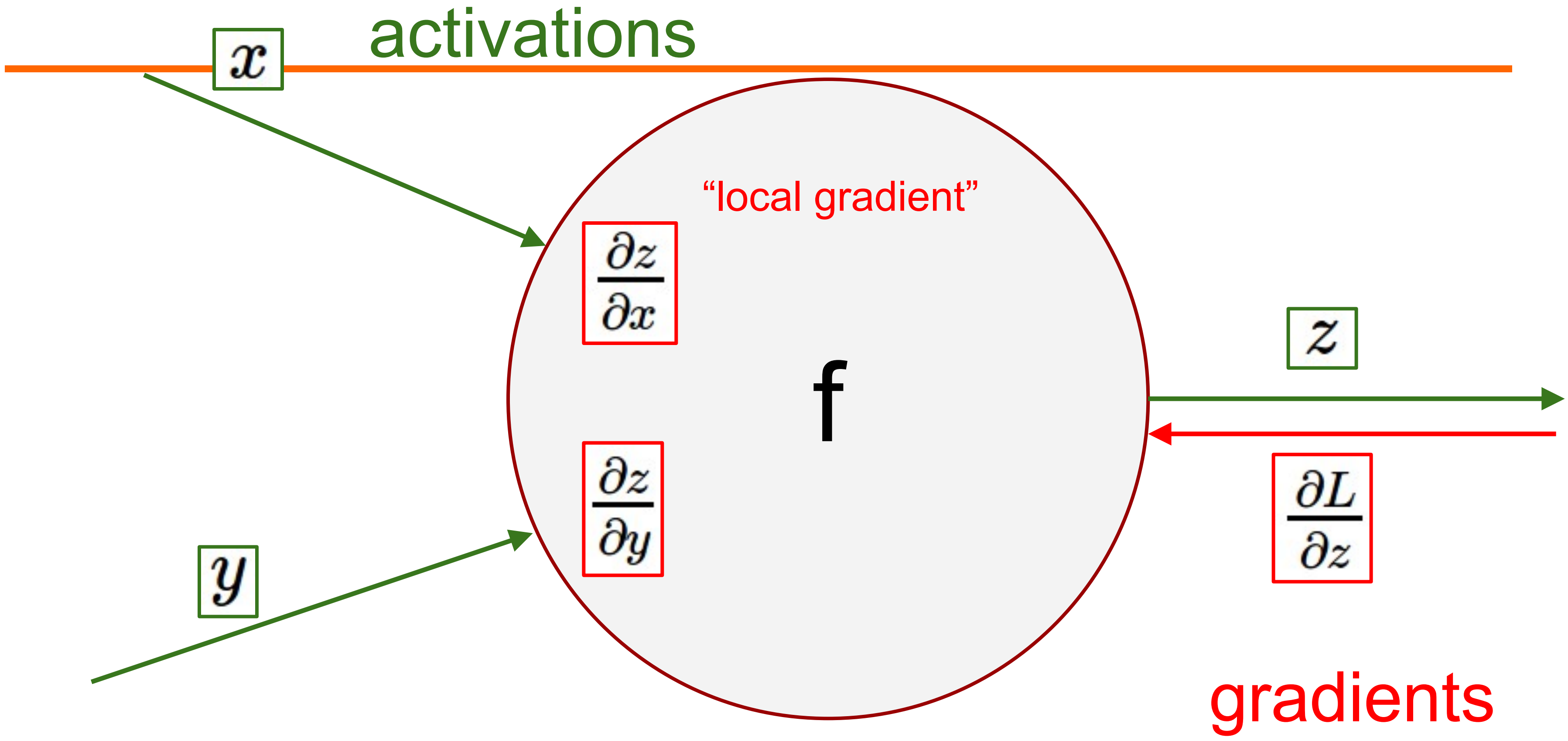

activations
x
f
z
y

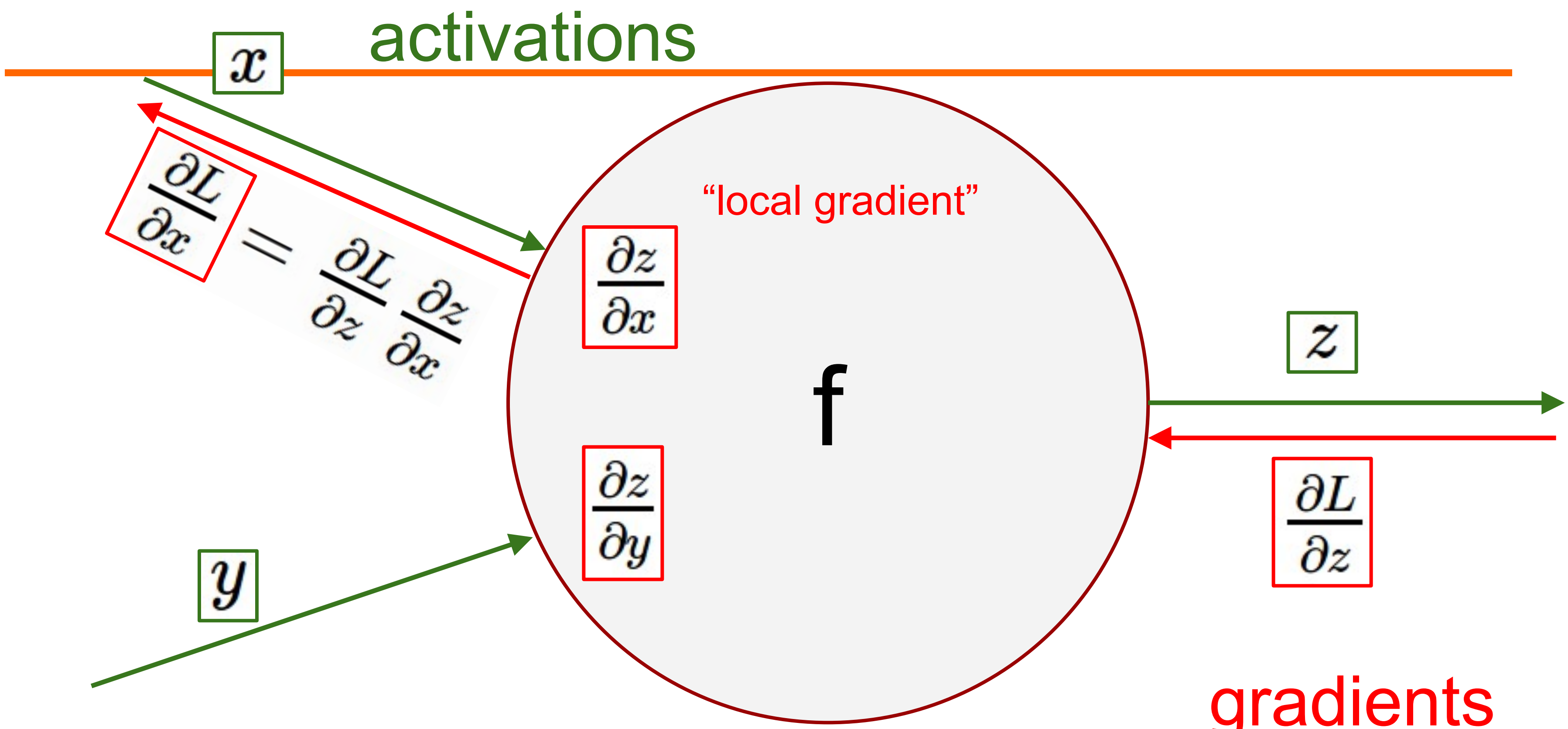


activations
x
"local gradient"
$\frac{\partial z}{\partial x}$
z
f
$\frac{\partial z}{\partial y}$
y

26


activations
x
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
gradients

27


activations
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$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
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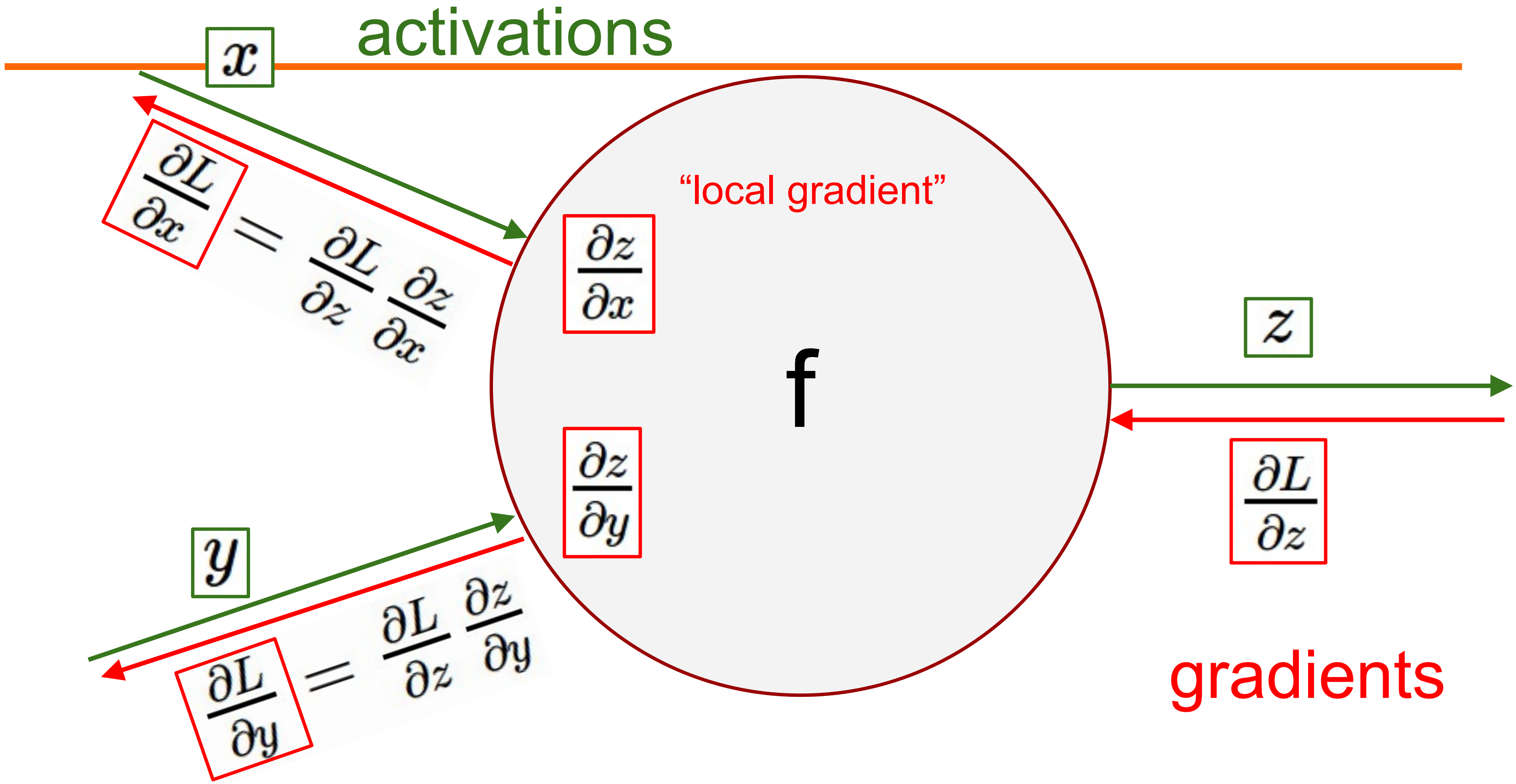
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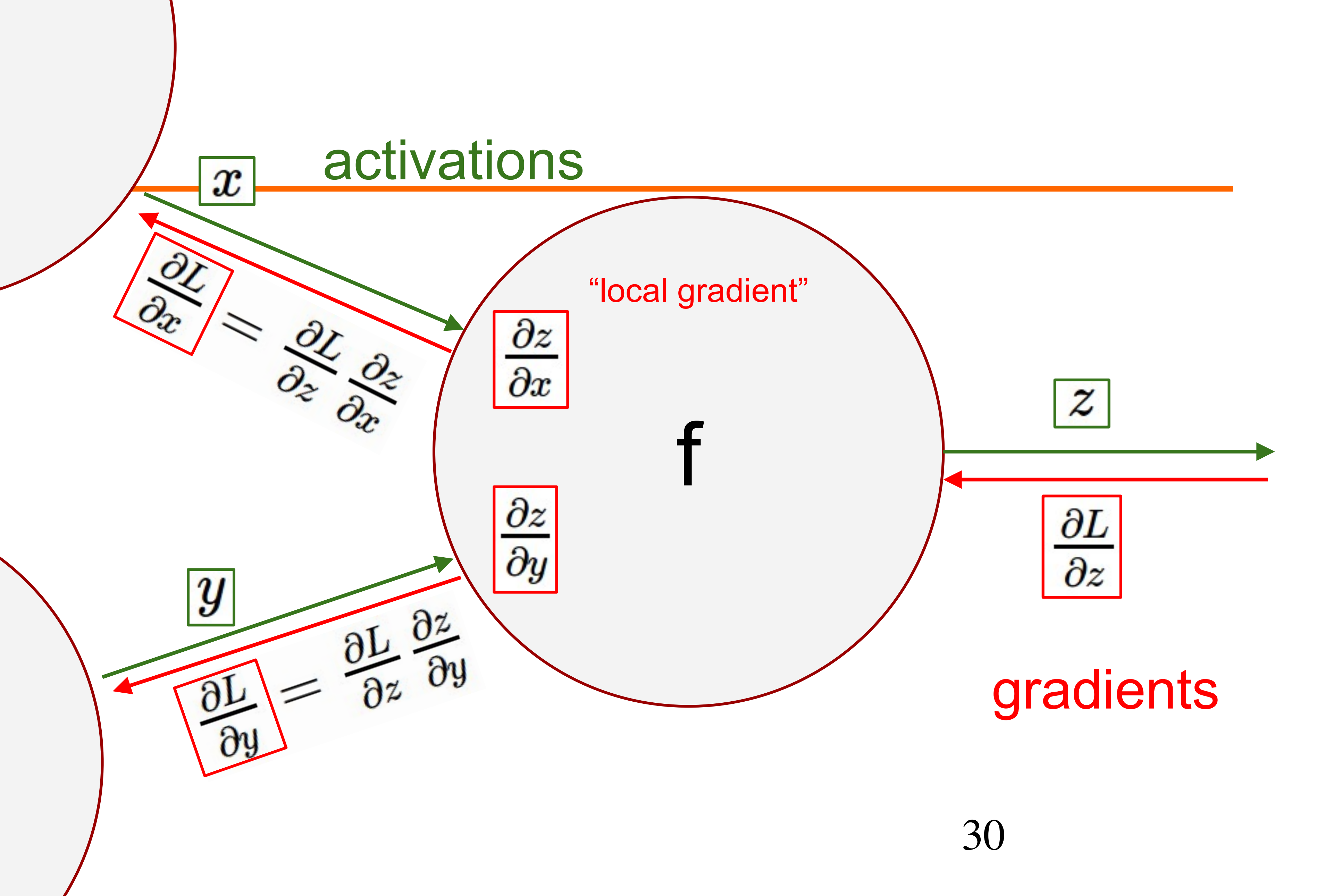
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y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
gradients

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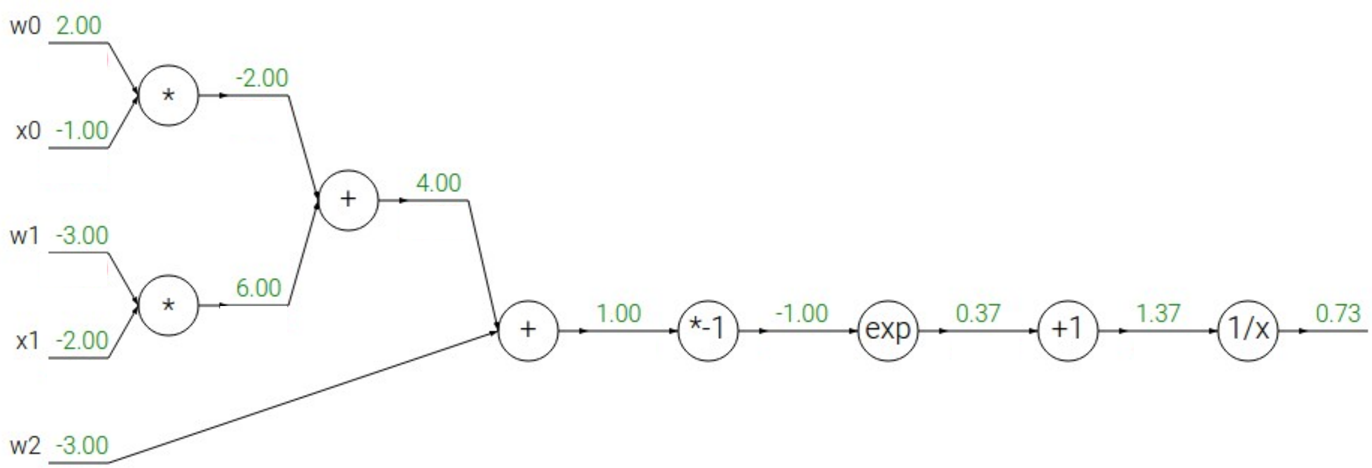
Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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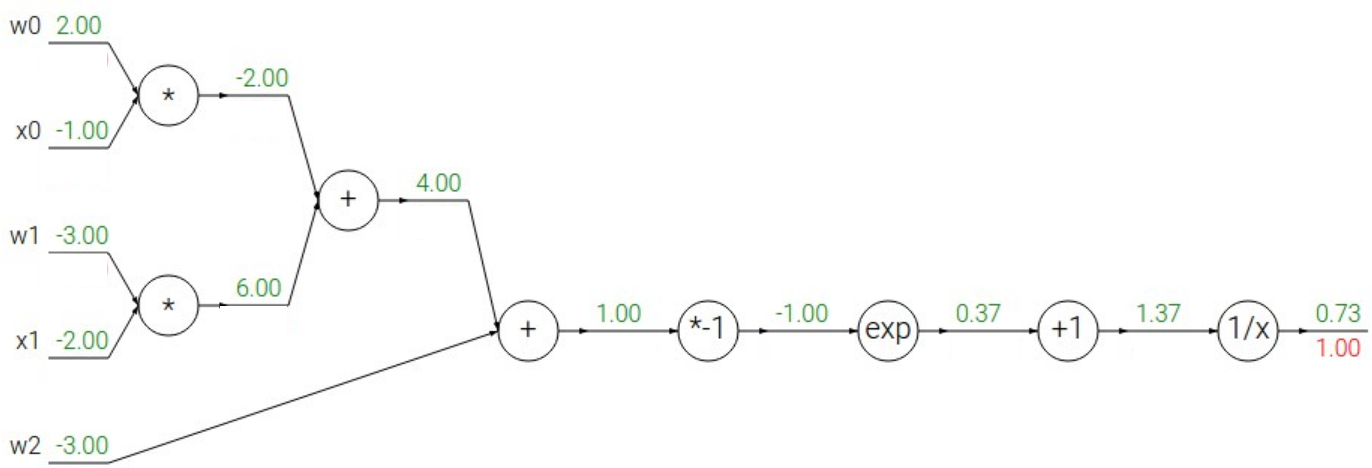

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

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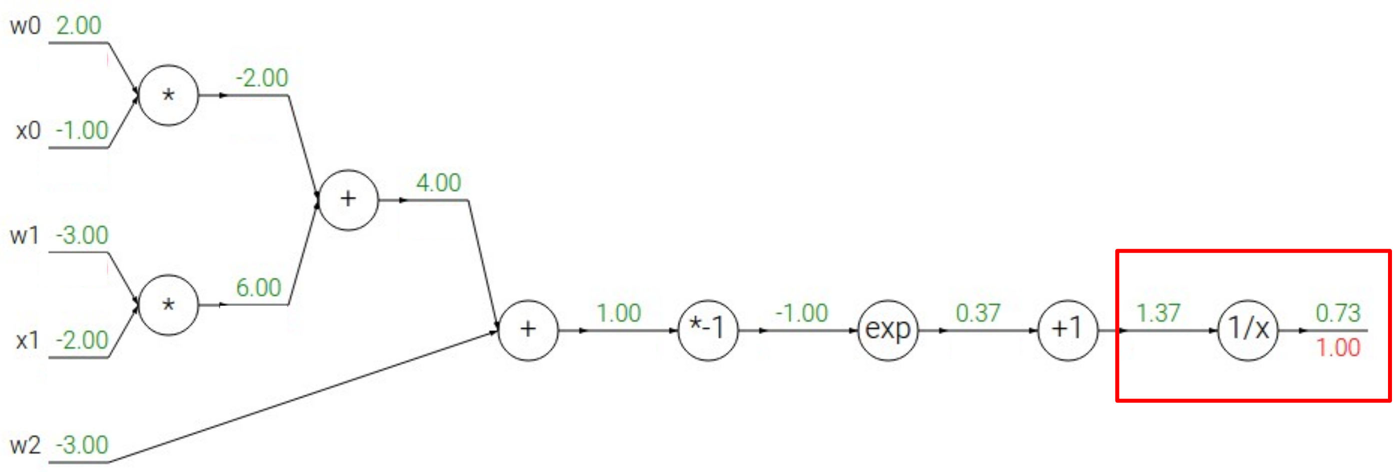

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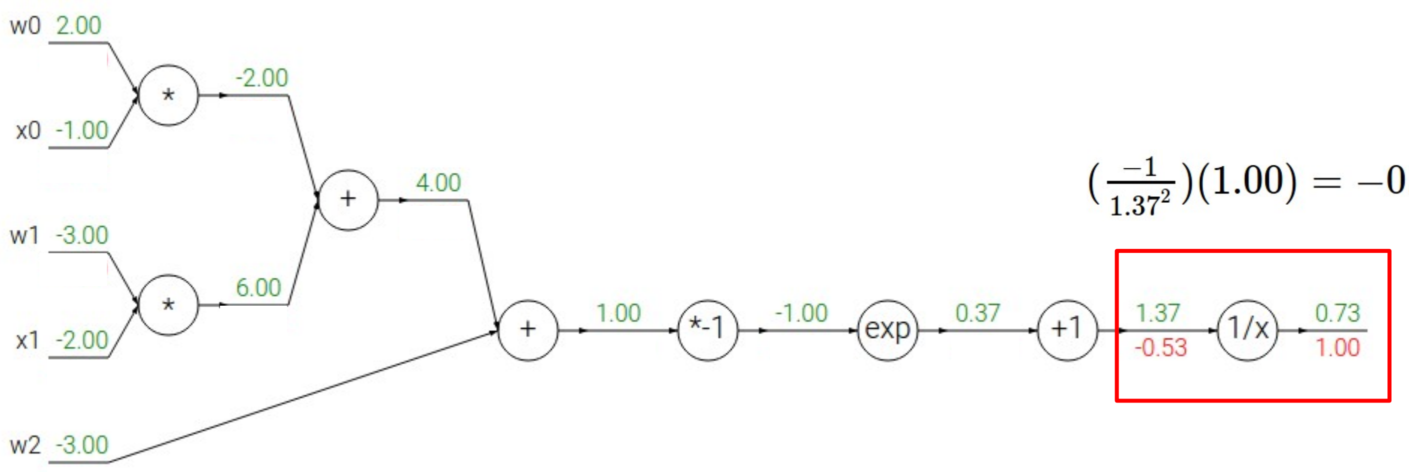

$$\left(\frac{-1}{1.37^2}\right)(1.00) = -0.53$$

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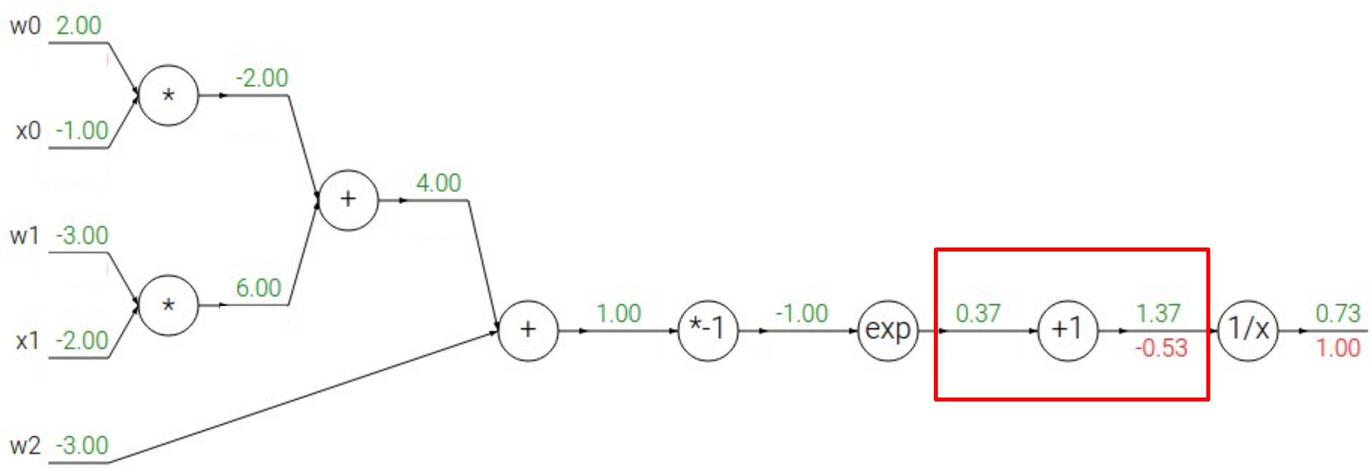

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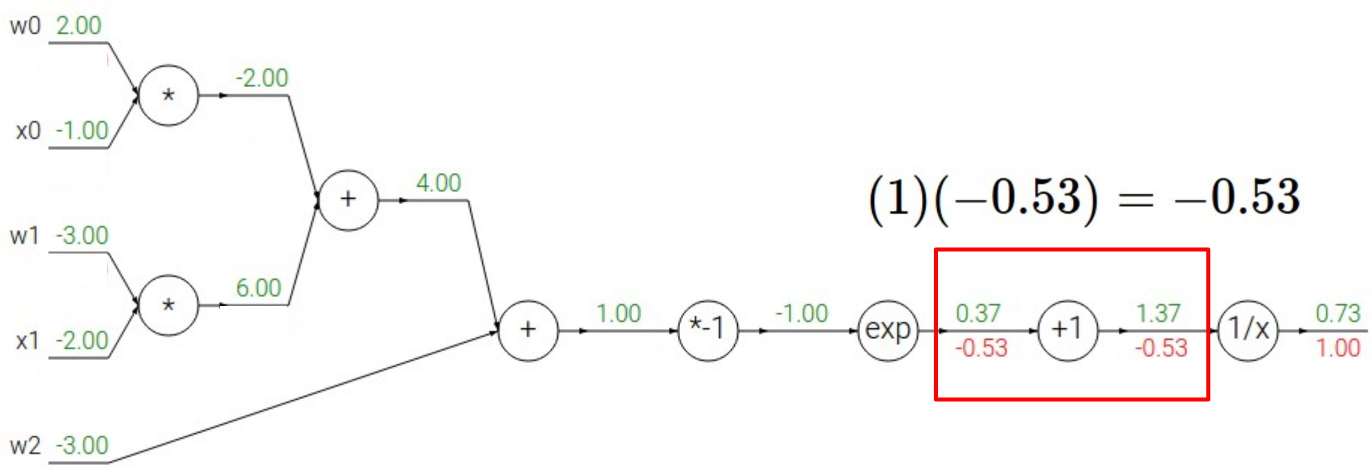

$(1)(-0.53) = -0.53$

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$

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36

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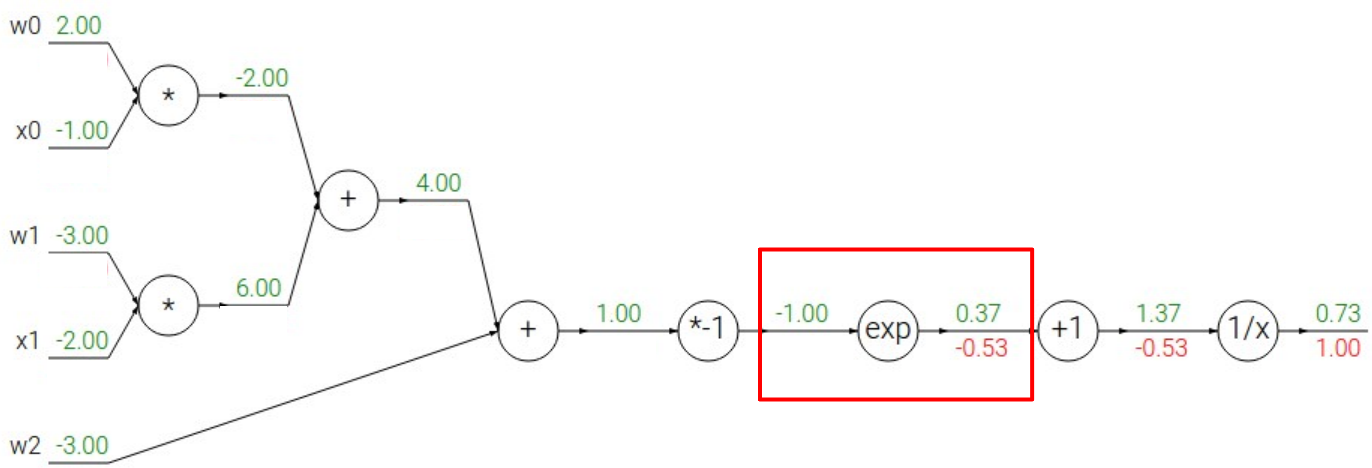

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

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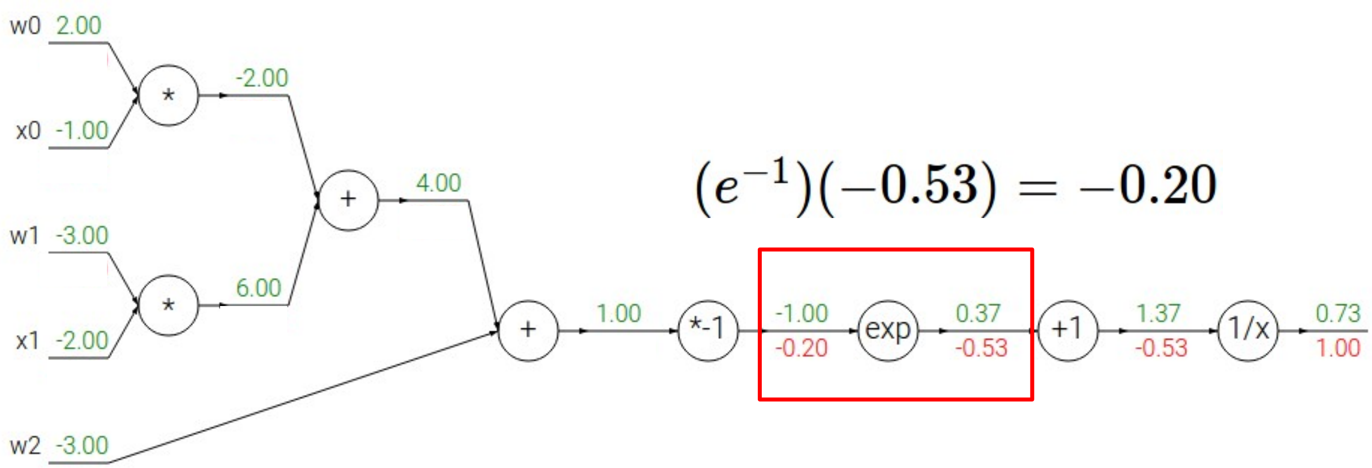
Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



$$(e^{-1})(-0.53) = -0.20$$

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

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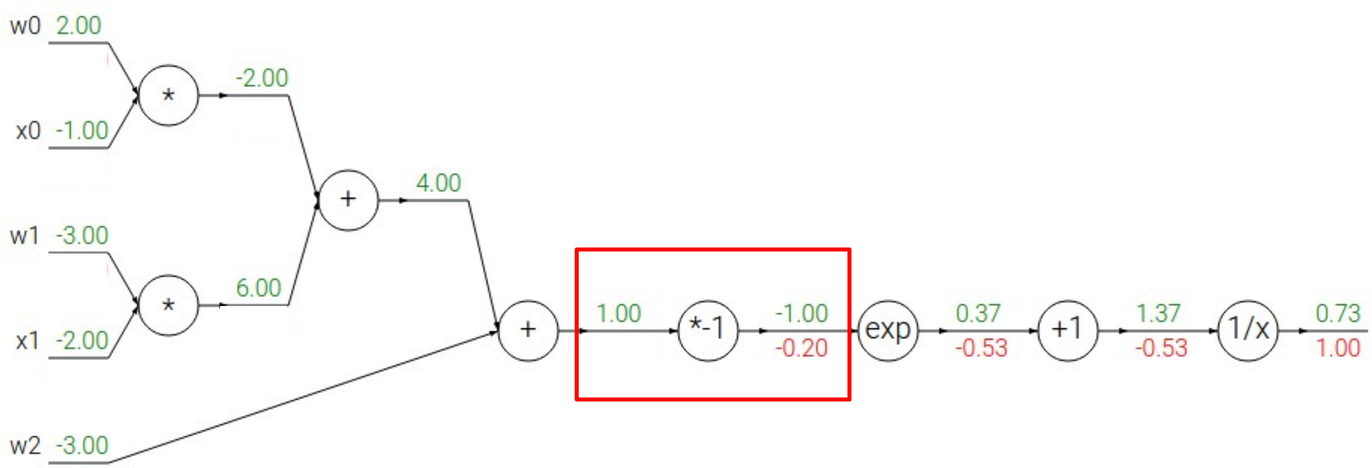

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$

$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$

$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$

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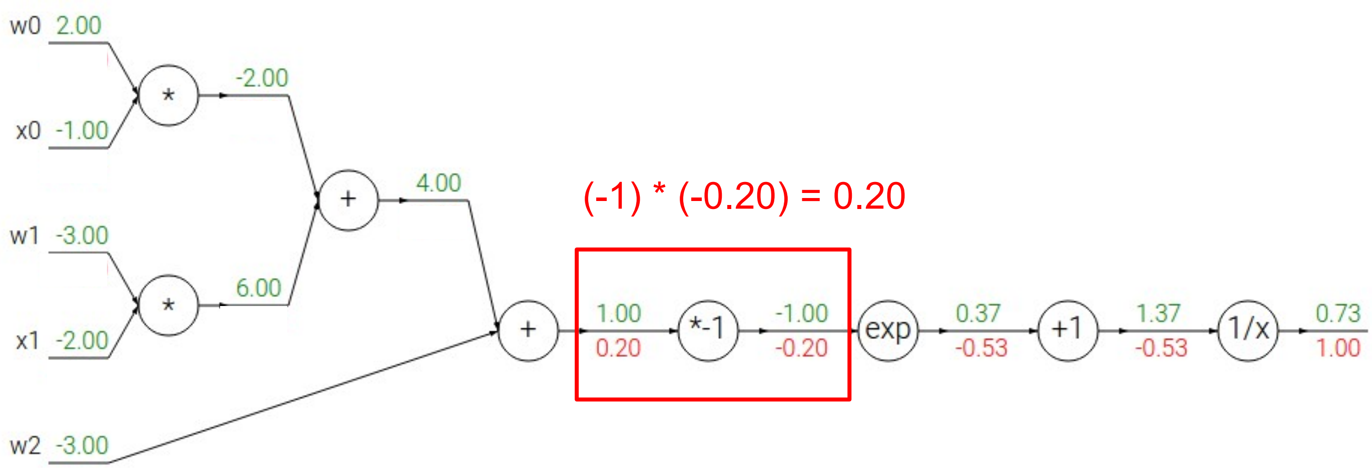

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

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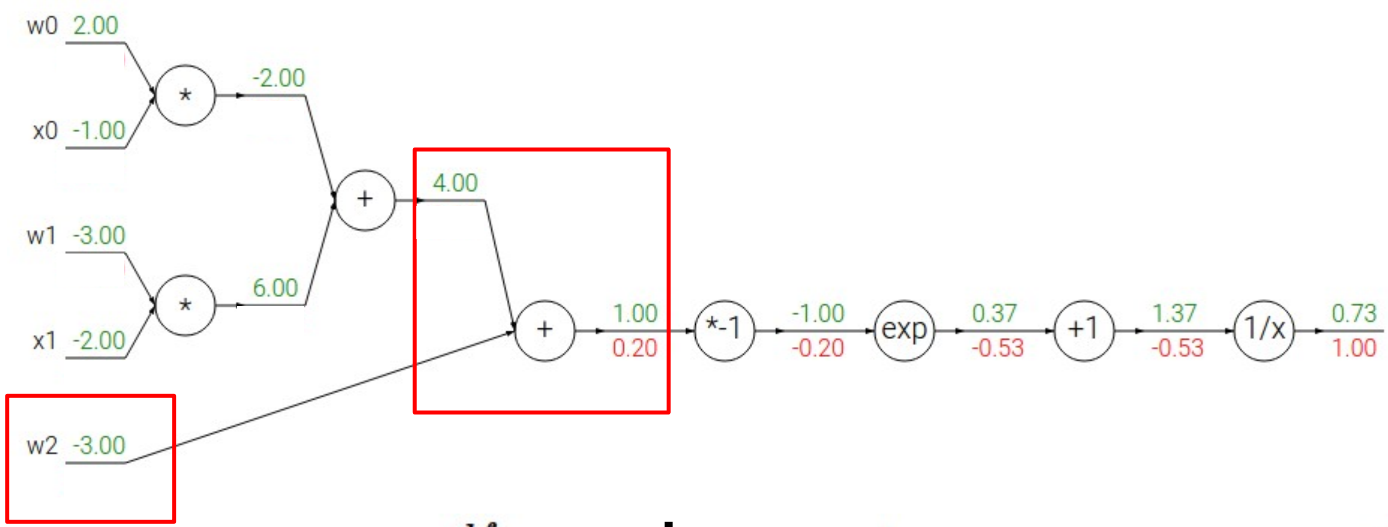
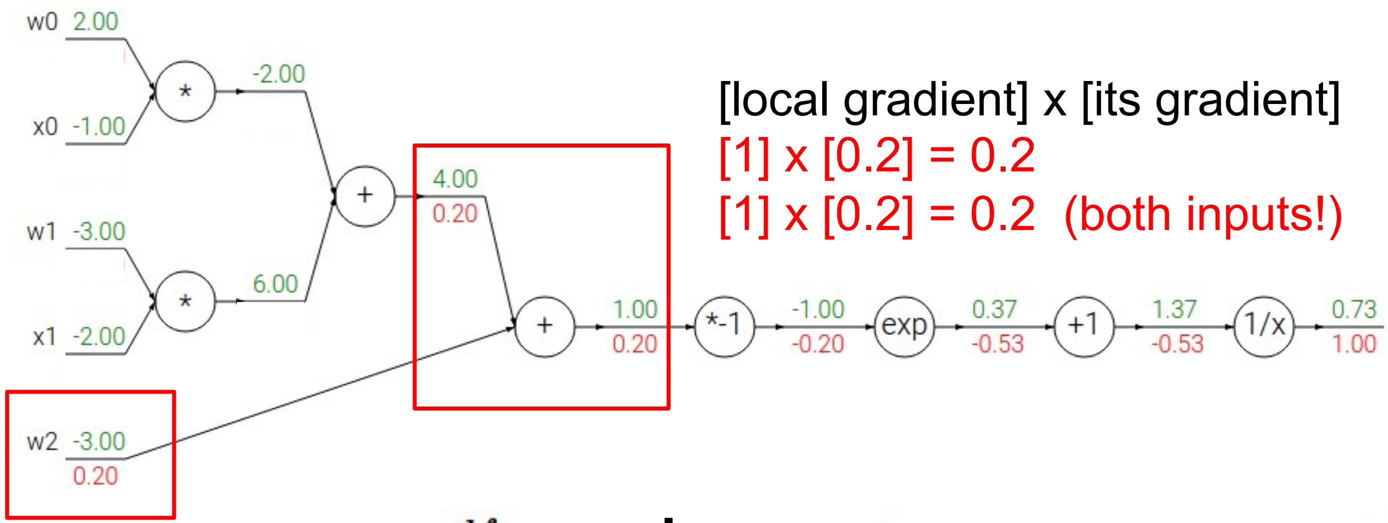
Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$		$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$	
$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$		$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$	

Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



[local gradient] x [its gradient]

[1] x [0.2] = 0.2

[1] x [0.2] = 0.2 (both inputs!)

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a \qquad\qquad f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

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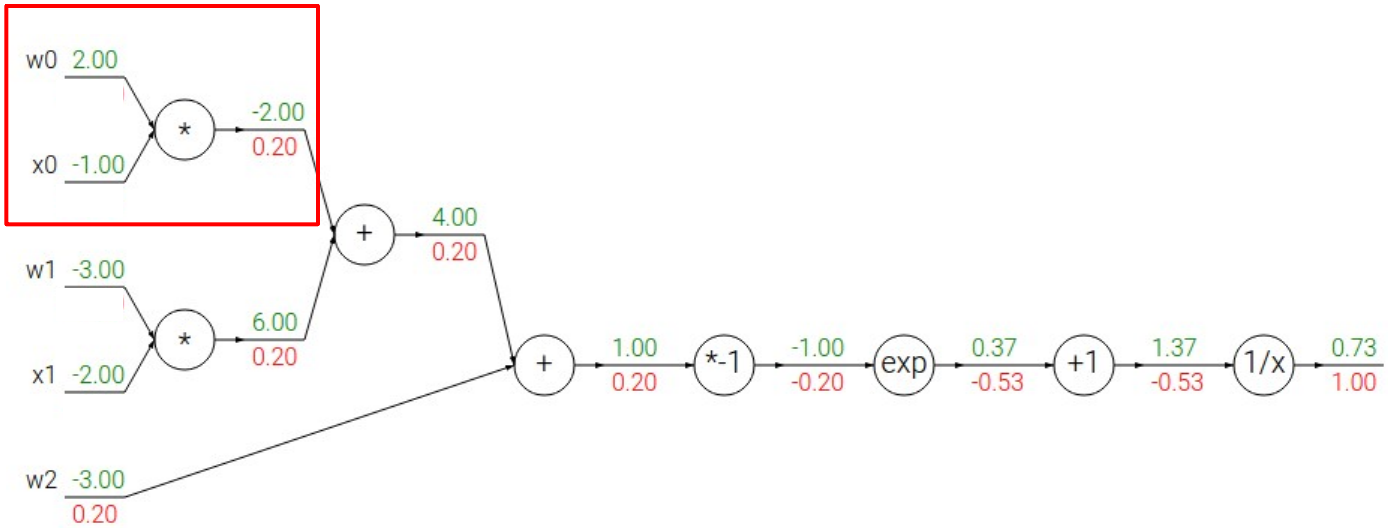
Another example:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



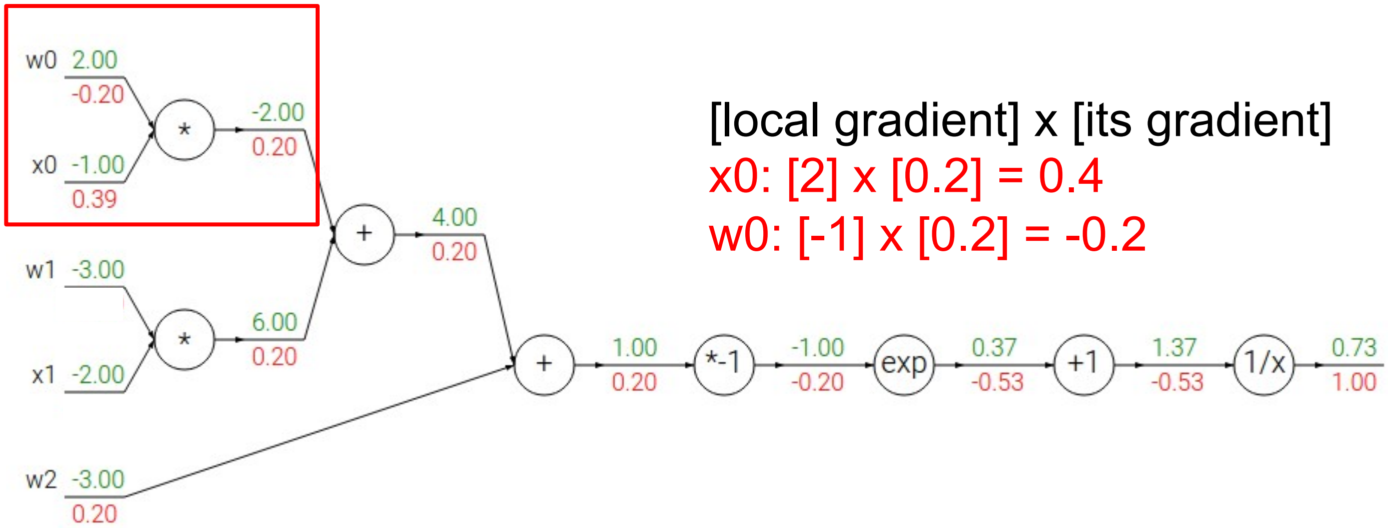
$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



[local gradient] x [its gradient]

x0: [2] x [0.2] = 0.4

w0: [-1] x [0.2] = -0.2

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

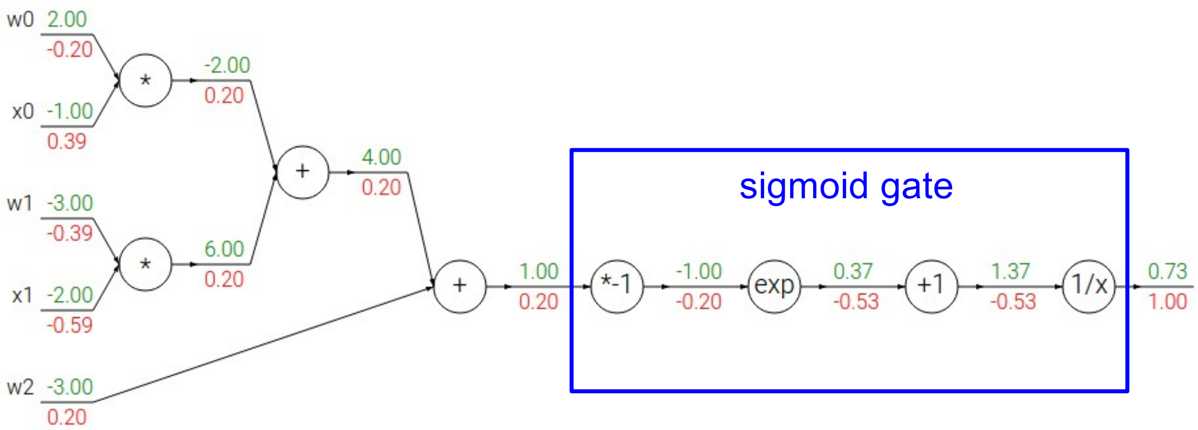
$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$



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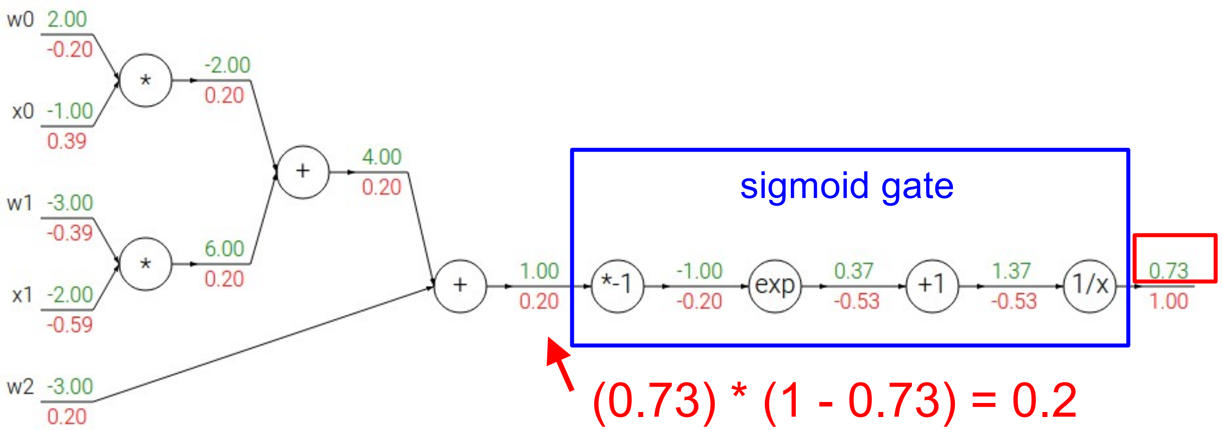
$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$



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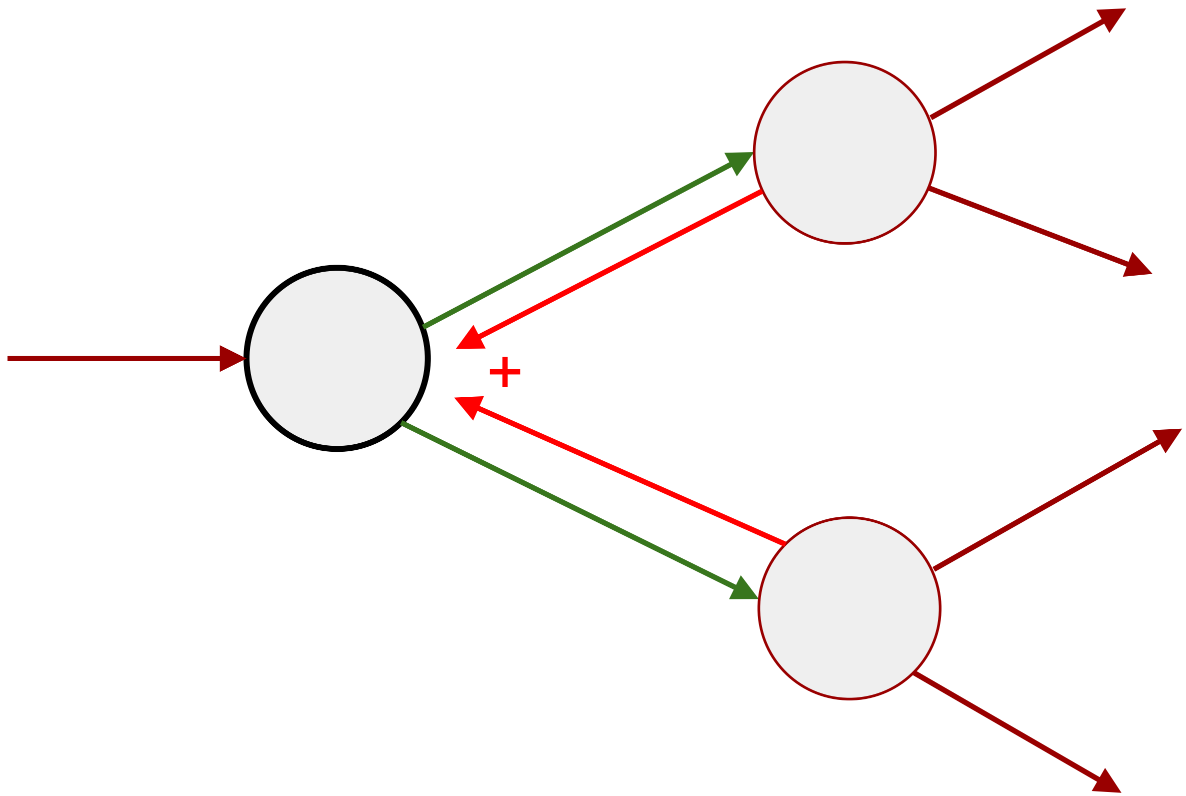
Gradients add at branches



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Summary

- Deep learning
  - New direction for test processing given its success in image/audio processing
  - Framworks and software
    - TensorFllow (Google).
    - Others: Theano, Torch, CAFFE, computation graph toolkit (CGT)