

Exercises in Quantum Computation VI

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Question 1. (Reading) Read “Physicists Triumph at Guess My Number”, at URL

<http://www.qubit.org/library/intros/gmn/gmn.html>.

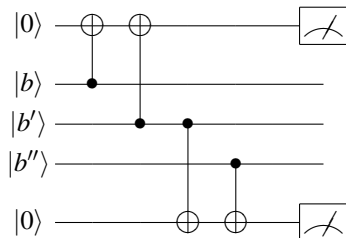
Question 2. (Classical Impossibilities) Consider the following distributed task that two parties A and B try to solve without mutual communication. Party A receives a random bit $a \in \{0, 1\}$ and B receives a random bit $b \in \{0, 1\}$. Given their respective inputs a and b , the goal is for A to produce a bit $x_a \in \{0, 1\}$ and for B to produce a bit $y_b \in \{0, 1\}$ such that $x_a \oplus y_b = ab$ for all possible inputs $(a, b) \in \{0, 1\}^2$. Prior to receiving the inputs a and b , the parties are allowed to discuss a joint strategy to solve this problem. But after the input bits have been distributed, A and B are no longer allowed to communicate.

(a) Find a mathematical proof that shows that it will be impossible for A and B to solve this problem with a 100% success rate.

(b) Design an approach that has a success rate that is higher than 50%.

(c) Assuming that A and B are only using classical methods, what is the highest success rate that they can achieve?

Question 3. (Towards Quantum Error Correction) Consider the following 5 qubit circuit



(a) If the two measurements have output “1” and “1”, what can you say about the input bits $(b, b', b'') \in \{0, 1\}^3$?

(b) In general, what do the two bits of the measurement outputs tell about the 3 input bits $(b, b', b'') \in \{0, 1\}^3$?

(c) Assume now that instead of the classical bits b, b', b'' , the three qubits in the middle are in a general superposition

$$|\psi\rangle = \sum_{xyz \in \{0,1\}^3} \alpha_{xyz} |x, y, z\rangle.$$

For which amplitudes α is the quantum state such that the two measurements do not affect the superposition $|\psi\rangle$?