Exercises in Quantum Computation II

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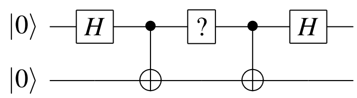
More exercises for the course "Quantum Computation and Quantum Information", Spring 2007.

Question 1. (The Effect of Pauli Gates). Consider the following 2 qubit circuit:



where the ?-gate can be one of the Pauli matrices $\{I,X,Y,Z\}$. Calculate what the output of this quantum circuit will be depending on the choice for the ?-gate.

Question 2. (Creating Correlated Quantum States). Describe a 4 qubit circuit that from the input $|0,0,0,0\rangle$ produces the state $\frac{1}{\sqrt{2}}(|0,0,0,0\rangle+|1,1,1,1\rangle)$.

Question 3. (Implementing Modulo Calculations). We are going to implement mod4 calculations in a unitary (and hence reversible) way. Let $a=(a_1,a_0)$ and $b=(b_1,b_0)$ represent two numbers $\in \mathbb{Z}_4$ according to $a=2a_1+a_0$ and $b=2b_1+b_0$. You are allowed to use all the standard gates.
(a) Describe a small 6 qubit circuit that implements the addition transformation $|a,b,0\rangle \mapsto |a,b,a+b \bmod 4\rangle$.
(b) Describe a small qubit circuit that implements the multiplication transformation $|a,b,0\rangle \mapsto |a,b,ab \bmod 4\rangle$.
(c) Contemplate how well you can generalize this to n bits numbers $a,b\in\mathbb{Z}_{2^n}$.

Question 4. (Copying Qubits). Notation: We define the qubit states $|+\rangle := \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ and $|-\rangle := \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$.
(a) The CNOT gate copies the bit value b in the operation $|b,0\rangle \mapsto |b,b\rangle$ for all $b\in\{0,1\}$. Describe a quantum circuit that copies the states $\{|+\rangle,|-\rangle\}$. That is: give a circuit that implements the mapping $|s,0\rangle \mapsto |s,s\rangle$ for all $s\in\{+,-\}$.
(b) Describe the effect of the circuit of the previous question on the states $|b,0\rangle$ for all $b\in\{0,1\}$.
(c) Prove that there exists no quantum circuit that implements a 2 qubit operation with the mappings $|0,0\rangle \mapsto |0,0\rangle$ and $|+,0\rangle \mapsto |+,+\rangle$. If possible, try to find more than one proof.

Question 5. (Exploring an Unknown Function). Take a Boolean function $f:\{0,1\}\to\{0,1\}$ and define its 2 qubit unitary implementation $U_f : |a,b\rangle \mapsto |a,b\oplus f(a)\rangle$.
(a) Apply this U_f to the superposition $\frac{1}{2}(|0\rangle+|1\rangle)\otimes(|0\rangle-|1\rangle)$ and see what happens to the two qubits depending on the function values $f(0)$ and $f(1)$.
(b) Ponder what you could do with the result of the previous question if you were allowed to apply some Hadamard gates after the application of U_f.

Acknowledgment: Again, the circuits in these exercises were drawn using the Q-circuit LaTeXpackage of Bryan Eastin and Steven T. Flammia.