Question 1 (Scalar products, vector products, and their meaning, 5+5 points).

(a) Prove the following equality: \( \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) \).

(b) Let the above vectors \( \mathbf{a} \), \( \mathbf{b} \) and \( \mathbf{c} \) define a three dimensional parallelepiped with vertices \( \pm \mathbf{a} \pm \mathbf{b} \pm \mathbf{c} \) (where each \( \pm \pm \pm \) combination defines one of the eight vertices). With this in mind, give a geometric interpretation of the above equality, including an interpretation of the vector products \( \mathbf{b} \times \mathbf{c} \) and \( \mathbf{c} \times \mathbf{a} \).

Question 2 (Basis independence of vector product, 10 points).

(a) Prove that the vector product \( \mathbf{r} \times \mathbf{s} \) is independent of the basis in which you express \( \mathbf{r} \), \( \mathbf{s} \) and \( \mathbf{r} \times \mathbf{s} \) (cf. Problem 1.16 for hints how to attack a similar question for the scalar product).

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Write the answers to the questions below on a separate set of pages.

Question 3 (Derivative of scalar product, 10 points).

(a) Prove that \( \frac{d}{dt}(\mathbf{r} \cdot \mathbf{s}) = \mathbf{r} \cdot \frac{d\mathbf{s}}{dt} + \frac{d\mathbf{r}}{dt} \cdot \mathbf{s} \).

Question 4 (Some differential equations, 5+5+5 points). Find the time dependent position \( x \) and velocity \( \dot{x} \) of a particle of mass \( m \) and with initial conditions \( x(t) = \dot{x}(t) = 0 \) at time \( t = 0 \), subject to the following force functions

(a) \( \mathbf{F} = (F_0 + \alpha t) \dot{x} \)

(b) \( \mathbf{F} = (F_0 + \beta x) \dot{x} \) (with \( \beta \geq 0 \))

(c) \( \mathbf{F} = (F_0 + \beta x) \dot{x} \) (with \( \beta \leq 0 \))

Question 5 (A question from Taylor, 5+5+5 points).

(a) Problem 1.36a

(b) Problem 1.36b

(c) Problem 1.36c