Classical Mechanics, Phys105A, Wim van Dam, UC Santa Barbara
Exercises Week 2; due Monday January 29, 11:30 am

Question 1 (Velocity with constant magnitude, 5+5 points). Let \( \mathbf{v}(t) \) be a time dependent vector describing the velocity of a particle moving in a 3 dimensional space. Prove the following two facts regarding the magnitude \( v = |\mathbf{v}| \) and the acceleration \( \mathbf{dv}/dt \).

\( \triangleright \) (a) As long as the acceleration \( \mathbf{dv}/dt \) is orthogonal to \( \mathbf{v} \), the magnitude \( v \) remains constant.

\( \triangleright \) (b) As long as the magnitude \( v \) remains constant, the acceleration \( \mathbf{dv}/dt \) has to be orthogonal to \( \mathbf{v} \).

Question 2 (A question from Taylor, 5+5 points).

\( \triangleright \) (a) Problem 1.46a

\( \triangleright \) (b) Problem 1.46b

Write the answers to the questions below on a separate set of pages.

Question 3 (Recovering the ‘dragless limit’, 5+5 points).

\( \triangleright \) (a) Consider the case of linear air resistance, described by the equation \( m\ddot{\mathbf{r}} = mg - b\mathbf{v} \). Prove that the results of Section 2.2 on the velocity and position of the particle coincide with the standard results on the movement of a particle moving in vacuum in the ‘dragless limit’ \( b \rightarrow 0 \).

\( \triangleright \) (b) For the case of quadratic air resistance with its equation \( m\ddot{\mathbf{r}} = mg - c|\mathbf{v}|\mathbf{v} \), answer the same question for the results on horizontal and vertical motion as derived in Section 2.4 in the dragless limit \( c \rightarrow 0 \).

Question 4 (Finding general solutions, 10 points).

\( \triangleright \) (a) Answer Taylor’s Problem 2.12.

Question 5 (A question from Taylor, 5+5 points).

\( \triangleright \) (a) Problem 2.54a

\( \triangleright \) (b) Problem 2.54b

Question 6 (Cubic drag, 5 + 5 points). Consider the case of horizontal motion with cubic drag, described by the equation \( m\ddot{x} = -cv^3 \).

\( \triangleright \) (a) Assuming initial speed \( v_0 \), derive the time dependency of the speed \( v \) of the particle.

\( \triangleright \) (b) Assuming initial speed \( v_0 \) and initial position \( x = 0 \), derive the time dependency of the position \( x \) of the particle.