Classical Mechanics, Phys105A, Wim van Dam, UC Santa Barbara
Exercises Week 2; due Monday January 29, 11:30 am

Question 1 (Velocity with constant magnitude, 5+5 points). Let $\mathbf{v}(t)$ be a time dependent vector describing the velocity of a particle moving in a 3 dimensional space. Prove the following two facts regarding the magnitude $v = |\mathbf{v}|$ and the acceleration $d\mathbf{v}/dt$.

▷ **(a)** As long as the acceleration $d\mathbf{v}/dt$ is orthogonal to $\mathbf{v}$, the magnitude v remains constant.

▷ **(b)** As long as the magnitude v remains constant, the acceleration $d\mathbf{v}/dt$ has to be orthogonal to $\mathbf{v}$.

Question 2 (A question from Taylor, 5+5 points).

▷ **(a)** Problem 1.46a

▷ **(b)** Problem 1.46b

Write the answers to the questions below on a separate set of pages.

Question 3 (Recovering the 'dragless limit', 5+5 points).

▷ **(a)** Consider the case of linear air resistance, described by the equation $m\ddot{\mathbf{r}} = m\mathbf{g} - b\mathbf{v}$. Prove that the results of Section 2.2 on the velocity and position of the particle coincide with the standard results on the movement of a particle moving in vacuum in the 'dragless limit' $b \to 0$.

▷ **(b)** For the case of quadratic air resistance with its equation $m\ddot{\mathbf{r}} = m\mathbf{g} - c|\mathbf{v}|\mathbf{v}$, answer the same question for the results on horizontal and vertical motion as derived in Section 2.4 in the dragless limit $c \to 0$.

Question 4 (Finding general solutions, 10 points).

▷ **(a)** Answer Taylor's Problem 2.12.

Question 5 (A question from Taylor, 5+5 points).

▷ **(a)** Problem 2.54a

▷ **(b)** Problem 2.54b

Question 6 (Cubic drag, 5 + 5 points). Consider the case of horizontal motion with cubic drag, described by the equation $m dv/dt = -cv^3$.

▷ **(a)** Assuming initial speed v_0, derive the time dependency of the speed v of the particle.

▷ **(b)** Assuming initial speed v_0 and initial position $x = 0$, derive the time dependency of the position x of the particle.