SOLUTION HW 1 PHYS 105 A

COMMENT TO mdalfonso@umail.ucsb.edu

$\boxed{1.a}$ $\hat{\imath}, \hat{\jmath}, \hat{k}$ vector basis

$$\vec{a} = a_1 \hat{\imath} + a_2 \hat{\jmath} + a_3 \hat{k}$$
$$\vec{b} = b_1 \hat{\imath} + b_2 \hat{\jmath} + b_3 \hat{k}$$
$$\vec{c} = c_1 \hat{\imath} + c_2 \hat{\jmath} + c_3 \hat{k}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot \Big[(b_2 c_3 - b_3 c_2)\hat{\imath} + (b_3 c_1 - b_1 c_3)\hat{\jmath} + (b_1 c_2 - b_2 c_1)\hat{k} \Big]$$

$$= a_1 b_2 c_3 - a_1 b_3 c_2 + a_2 b_3 c_1 - a_2 b_1 c_3 + a_3 b_1 c_2 - a_3 b_2 c_1$$

$$= b_1 (a_3 c_2 - a_2 c_3) + b_2 (a_1 c_3 - a_3 c_1) + b_3 (a_2 c_1 - a_1 c_2)$$

$$= \vec{b} \cdot (\vec{c} \times \vec{a})$$

1.b

The cross product $\vec{b} \times \vec{c}$ correspond to the area of the parallelogram with sides $\vec{b}$ and $\vec{c}$ ~~xxxx~~

When we multiply $\vec{a}$ and $\vec{b} \times \vec{c}$ we get the volume of parallelepiped.

The ~~x~~ relation found in problem 1.a

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} (\vec{c} \times \vec{a})$$

Says that the volume is the same if you multiply the area of $\vec{b} \times \vec{c}$ with vector $\vec{a}$ or if you multiply the area of $\vec{c} \times \vec{a}$ with vector $\vec{b}$.

$\boxed{2}$

Consider an ortogonal basis $\hat{i}, \hat{j}, \hat{k}$ and another basis $\hat{i}', \hat{j}', \hat{k}'$ and a rotation R to transform from one basis to another.

Let $\vec{r}$ be ~~a~~ vector as described in basis $\hat{i}, \hat{j}, \hat{k}$ and $\vec{r}' = R\vec{r}$ is the vector the described in basis $\hat{i}', \hat{j}', \hat{k}'$

The length of vector is ~~...~~ independent from reference systems used

Transpost ~~of~~

$$|\vec{r}'|^2 = |R\vec{r}|^2 = (R\vec{r})^T (R\vec{r}) = \vec{r}^T R^T R \vec{r}$$

because of ~~...~~ R is a rotation $R^T R = I$

$$\Rightarrow |\vec{r}'|^2 = |\vec{r}\,\vec{r}| = |\vec{r}|^2$$

~~It follows that also that any s...~~

Now given $\vec{r}, \vec{s}$

the magnitude $|\vec{r} \times \vec{s}| = \sqrt{|\vec{r}|^2 |\vec{s}|^2 - (\vec{r} \cdot \vec{s})^2}$

$|\vec{r}|^2$ and $|\vec{s}|^2$ are the lenght of vector $\vec{r}$ and $\vec{s}$

and $(\vec{r} \cdot \vec{s})$ ~~and~~ is invariant under rotation also

(can be demonstrated as we demonstrate that $|\vec{r}|^2 = |\vec{r}'|^2$

in

$\Rightarrow |\vec{r} \times \vec{s}|$ is independent froma the basis used to describe $\vec{r}, \vec{s}, \vec{r} \times \vec{s}$

the <u>direction</u> of vector $\vec{r} \times \vec{s}$ is always perpendicular to the plane made of vector $\vec{r}$ and $\vec{s}$ and follow the right hand rule.

$\Rightarrow$ also the direction of $\vec{r} \times \vec{s}$ is independent from the reference system used to describe $\vec{r}$, $\vec{s}$, $\vec{r} \times \vec{s}$.

Solution to HW # 1 (Phys 105A)
(Part II)

Question 3

$$\vec{r}\cdot\vec{s} = r_x s_x + r_y s_y + r_z s_z$$

$$\frac{d}{dt}(r_x s_x) = \frac{dr_x}{dt} s_x + r_x \frac{ds_x}{dt} \quad (\because \text{chain rule})$$

similary. for $r_y s_y$, $r_z s_z$.

So $$\frac{d(\vec{r}\cdot\vec{s})}{dt} = \frac{dr_x}{dt} s_x + \frac{dr_y}{dt} s_y + \frac{dr_z}{dt} s_z + r_x \frac{ds_x}{dt} + r_y \frac{ds_y}{dt} + r_z \frac{ds_z}{dt}$$

$$= \underline{\underline{\frac{d\vec{r}}{dt}\cdot\vec{s} + \vec{r}\cdot\frac{d\vec{s}}{dt}}}$$

Question 4

(a) $\vec{F} = (F_0 + \alpha t)\hat{x} = m\frac{d^2x}{dt^2}\hat{x}$ $\Rightarrow$ integrate over t

$$\Rightarrow \int_0^t (F_0 + \alpha t)\, dt = m\left(\dot{x}(t) - \dot{x}(0)\right)$$

$$= F_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \dot{x}(t) = \underbrace{\dot{x}(0)}_{(=0)} + \frac{F_0}{m}t + \frac{\alpha}{2m}t^2 = \frac{F_0 t}{m} + \frac{\alpha t^2}{2m}$$

integrate over t

$$x(t) = \underline{\underline{\frac{F_0 t^2}{2m} + \frac{\alpha t^3}{6m}}} \quad (\because x(0) = 0)$$

(b) $\vec{F} = (F_0 + \beta x)\hat{x} = m\ddot{x}\,\hat{x}$, $\beta \geq 0$

$$\Rightarrow \quad \ddot{x} - \frac{\beta}{m} x = \frac{F_0}{m}, \quad \text{let} \quad \frac{\beta}{m} = \omega^2 > 0$$

then. $\ddot{x} - \omega^2 x = \frac{F_0}{m}$.

A general solution to this differential equation is

$$x(t) = \underbrace{A e^{\omega t} + B e^{-\omega t}}_{\text{(homogeneous sol)}} \; \underbrace{- \frac{F_0}{m\omega^2}}_{\text{(inhomogeneous sol)}}$$

and our initial condition is $x(0) = \dot{x}(0) = 0$

$$\text{so,} \quad x(t) = \frac{F_0}{2m\omega^2}\left(e^{\omega t} + e^{-\omega t}\right) - \frac{F_0}{m\omega^2}$$

$$= \frac{F_0}{m\omega^2}\left(\cosh \omega t - 1\right)$$

$$= \frac{F_0}{\beta}\left(\cosh \sqrt{\frac{\beta}{m}}\, t - 1\right)$$

(b) $\vec{F} = (F_0 + \beta x)\hat{x} = m\ddot{x}\hat{x}$

$$\ddot{x} - \frac{\beta}{m} x = \frac{F_0}{m}. \quad \text{let} \quad -\frac{\beta}{m} = \omega^2 > 0$$

then $\ddot{x} + \omega^2 x = \frac{F_0}{m}$

$\Rightarrow$ general sol, $\Rightarrow$ $x = A\cos\omega t + B\sin\omega t + \frac{F_0}{m\omega^2}$

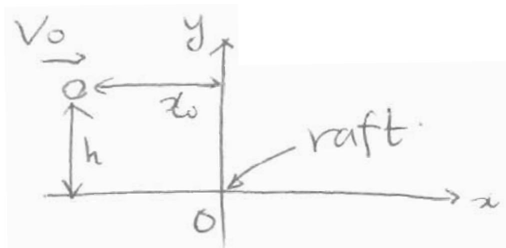
implementing initial condition $(x(0) = \dot{x}(0) = 0)$

$$\Rightarrow x(t) = \frac{F_0}{m\omega^2}(1 - \cos\omega t)$$

or. $x(t) = -\frac{F_0}{\beta}\left(1 - \cos\sqrt{\frac{-\beta}{m}}\,t\right)$

$$= \underline{\frac{F_0}{\beta}\left(\cos\sqrt{\frac{-\beta}{m}}\,t - 1\right)}$$

Question 5)



(a)

Newton's law $\Rightarrow m\ddot{\vec{x}} = \vec{F} = -mg\,\hat{y}$

$$\Rightarrow \begin{cases} x(t) = x_0 + V_0 t \\ y(t) = h - \frac{1}{2}gt^2 \end{cases}$$ (trajectory of a bundle)

(b) when $y(t) = 0 \Rightarrow t = \sqrt{\frac{2h}{g}} = t_*$

and $x(t_*) = x_0 + V_0\sqrt{\frac{2h}{g}} = 0$ (to reach a raft)

$\Rightarrow x_0 = -V_0\sqrt{\frac{2h}{g}}$. or $|x_0| = V_0\sqrt{\frac{2h}{g}}$

$= 50 \cdot \sqrt{20}\ m$

$\underline{\cong 223\ m}$

(c) $\cancel{|x_0| = V_0 t}$ $\Rightarrow$ $-\Delta x_0 = V_0 \Delta t$, $\Delta x_0 = \pm 10\,m$

$-x_0 = V_0 t$

$\Delta t = \pm \frac{10\,m}{50\,m/s} = \underline{\pm 0.2\,sec}$