Name: Perm. No.

# Midterm Phys105A, Thursday February 15, 2007, 9:30–10:45 (70 points)

**Question 1** (20 points). Time of impact under inverse quadratic force

We drop a particle with mass $m$ at distance $r = d$ from the origin under the influence of a central potential $U(r) = -km/r$. Let $s$ be the time required for the particle to reach the origin $r = 0$. As a function of $m$ and $d$, it holds that $s = \gamma m^\alpha d^\beta$.

▷ **(a)** Determine these powers $\alpha$ and $\beta$.

**Question 2** (10+10 points). A Nonconservative Force

Consider a nonconservative force defined over the plane with the following (topological) property for the work done over the closed paths from 1 back to 1: $W(1 \to 1) = 0$ if the loop does not go around the origin $O$, $W(1 \to 1) = c$ if the loop goes around the origin $O$ once in a clockwise fashion, $W(1 \to 1) = -c$ if the loop goes around the origin $O$ once in an anti-clockwise fashion, and so on. In other words $W(1 \to 1)/c$ counts how many times the path went around $O$ clockwise.

▷ **(a)** Write down a force $\mathbf{F}$ that has this property. Give arguments why your answer is correct.

▷ **(b)** Locally, in small patches that does not involve the origin, this force is conservative, and we can indeed give a local potential like function $V(r, \phi)$ with all the right properties. Yet globally no such potential should exist. What is going on here?

**Question 3** (10+20 points). Orbits and Central Forces

A particle with mass $m$ moves in the plane under influence of a central force $f(r)\hat{e}_r$. The trajectory of the particle is described by $r(t) = r_0 \mathrm{e}^{k\cdot\phi(t)}$ where $\phi(t)$ is the time dependent angle in the polar coordinate system that we are using.

▷ **(a)** Prove that $\phi(t)$ has to change logarithmically in time $t$.

▷ **(b)** Prove that $f(r)$ has to depend in an inverse cube way on $r$.