Classical Mechanics

Phys105A, Winter 2007

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Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…
- Homework 3 has been posted.
  It is due Monday January 29, 11:30 am.
  Be sure to download the ‘improved’ version.
- Midterm is scheduled for Week 6, Thursday Feb 15.
  The material will be Chapters 1–4; it is not open book, but you are allowed a ‘cheat sheet’.

Force as a Gradient of *U*

Using the *gradient* we can summarize the connection between a potential U and its corresponding conservative force **F** by the statement:

$$\mathbf{F} = -\nabla U = -\frac{\partial U}{\partial x}\hat{e}_x - \frac{\partial U}{\partial y}\hat{e}_y - \frac{\partial U}{\partial z}\hat{e}_z$$

Note that each possible potential U gives rise to a conservative force $\mathbf{F} = -\nabla U$.

The “Del” Operator ∇

You can think of del as $\nabla = \partial/\partial x + \partial/\partial y + \partial/\partial z$.

The *gradient* ∇U of a scalar field U is a vector field:

$$\nabla U = \partial U/\partial x \, \mathbf{e}_x + \partial U/\partial y \, \mathbf{e}_y + \partial U/\partial z \, \mathbf{e}_z.$$

The *curl* $\nabla\times\mathbf{V}$ of a vector field **V** is a vector field:

$$\nabla\times\mathbf{V} = (\partial V_z/\partial y - \partial V_y/\partial z)\, \mathbf{e}_x + (\partial V_x/\partial z - \partial V_z/\partial x)\, \mathbf{e}_y + (\partial V_y/\partial x - \partial V_x/\partial y)\, \mathbf{e}_z.$$

The *divergence* $\nabla\cdot\mathbf{V}$ of a vector field **V** is a scalar field:

$$\nabla\cdot\mathbf{V} = \partial V_x/\partial x + \partial V_y/\partial y + \partial V_z/\partial z.$$

$\nabla \times \mathbf{F} = 0 \Leftrightarrow$ F is Conservative

The fact that F is a conservative force is equivalent with the statement that $\nabla \times F = 0$ at all positions (x,y,z).

To prove this we use Stokes' Theorem:

$$\oint_{\Gamma} \mathbf{F} \bullet d\mathbf{r} = \int_{\Sigma} \nabla \times \mathbf{F} \bullet d\boldsymbol{\sigma}$$

where on the left hand we integrate over the boundary Γ of the surface Σ, while on the right hand we integrate over the surface itself (using the normal vector $d\boldsymbol{\sigma} = \mathbf{e}_n \bullet d\sigma$).

Understanding Stokes' Thm

To understand Stokes' theorem is it helps to know that for small squares ($\mathbf{r},\mathbf{r}+\Delta x,\mathbf{r}+\Delta y,\mathbf{r}+\Delta x+\Delta y$) we have at $\mathbf{r}$:

$$\nabla \times \mathbf{F} = \frac{1}{\Delta x\ \Delta y} \oint_{r \to r+\Delta x \to r+\Delta x+\Delta y \to r+\Delta y \to r} \mathbf{F} \bullet d\mathbf{r}' \qquad \text{as } \Delta x, \Delta y \to 0.$$

By patching such infinitesimal squares we can make proper surfaces, thus obtaining Stokes' Theorem.

The requirement $\nabla\times\mathbf{F}=\mathbf{0}$ certifies that each (infinitesimal) circular path will have a net work $W(\mathbf{r}\to\mathbf{r}) = 0$.

Time Dependent U

- For time dependent U, we have at any given moment that $\mathbf{F} = -\nabla \mathbf{U}$ is conservative, yet the total energy $E = T+U$ is not conserved.
- $dT \;= m(\mathbf{v}\bullet d\mathbf{v}) = m(\mathbf{v}\bullet\mathbf{a})dt = \mathbf{F}\bullet d\mathbf{r}$
- $dU \;= \partial U/\partial x\, dx + \partial U/\partial y\, dy + \partial U/\partial z\, dz + \partial U/\partial t\, dt$
  $= \nabla \mathbf{U}\bullet d\mathbf{r} + \partial U/\partial t\, dt = -\mathbf{F}\bullet d\mathbf{r} + \partial U/\partial t\, dt$

- So, $dE = d(T+U) = \partial U/\partial t\, dt$ and $\Delta E = \int \partial U/\partial t\, dt$.