Classical Mechanics

Phys105A, Winter 2007

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Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…
- Homework 4 has been posted.
  It is due Monday February 12, 11:30 am.
- Midterm is scheduled for Week 6, Thursday Feb 15.
  The material will be Chapters 1–4; no electronics are allowed, it is not open book, but you are allowed a letter sized, double sided ‘cheat sheet’.

Energy of Two Particles

For two particles under the influence of conservative forces, the total energy E is again the sum of kinetic energy and potential energy.

The kinetic energy is straightforward: $T = T_1+T_2$ with $T_1 = ½\, m_1\, v_1^2$ and $T_2 = ½\, m_2\, v_2^2$. Similarly we have for the external potential: $U^{ext} = U_1^{ext} + U_2^{ext}$.

More subtle is the potential energy U^{int} due to the interacting forces between the particles.

Can we give a potential for this?

Potential between Particles

To simplify matters, assume that there is no external force.

- The force on (1) exerted by (2) is $\mathbf{F}_{12}$, similarly for $\mathbf{F}_{21}$.
- By Newton's 3rd law $\mathbf{F}_{12} = -\mathbf{F}_{21}$.
- Assume that the force depends only on the positions of the particles, hence $\mathbf{F}_{12} = \mathbf{F}_{12}(\mathbf{r}_1,\mathbf{r}_2)$.
- If **F** is *translationally invariant* we have $\mathbf{F}_{12} = \mathbf{F}_{12}(\mathbf{r}_1-\mathbf{r}_2)$.

With $\mathbf{r}_1=(x_1 y_1,z_1)$ and $\mathbf{r}_2=(x_2,y_2,z_2)$ we can define the differential operator $\nabla_1 = \partial/\partial x_1+\partial/\partial y_1+\partial/\partial z_1$, and $\nabla_2 = \ldots$

If $\mathbf{F}_{12}$ is a conservative force we have $\nabla_1\times\mathbf{F}_{12} = \mathbf{0}$ and there is a potential U such that $\mathbf{F}_{12} = -\nabla_1 U(\mathbf{r}_1-\mathbf{r}_2)$.

For (2) we have $\mathbf{F}_{21} = -\mathbf{F}_{12} = \nabla_1 U(\mathbf{r}_1-\mathbf{r}_2) = -\nabla_2 U(\mathbf{r}_1-\mathbf{r}_2)$.

One Potential, Two Particles

The previous slide shows that for (1) and (2) we have one potential $U(\mathbf{r}_1 - \mathbf{r}_2)$ on the position of (1) relative to the position of (2) such that $\mathbf{F}_{12} = -\nabla_1 U$ and $\mathbf{F}_{21} = -\nabla_2 U$. Note that for (2) we do *not* use $U(\mathbf{r}_2 - \mathbf{r}_1)$.

As the particles move, the work dT done is now $dT = dT_1 + dT_2 = d\mathbf{r}_1 \cdot \mathbf{F}_{12} + d\mathbf{r}_2 \cdot \mathbf{F}_{21} = (d\mathbf{r}_1 - d\mathbf{r}_2) \cdot \mathbf{F}_{12}$.

Hence indeed $dT = d(\mathbf{r}_1 - \mathbf{r}_2) \cdot [-\nabla_1 U(\mathbf{r}_1 - \mathbf{r}_2)] = -dU$ and the total energy $E = T_1 + T_2 + U$ stays conserved: $dE = dT + dU = 0$.

Important: For the potential energy of two particles you have only one $U(\mathbf{r}_1 - \mathbf{r}_2)$, *not* $U_1 + U_2$ or so.

Elastic Collisions

A collision between two particles is in conservative interaction between (1) and (2) with **F** going to **0** as the relative distance $|\mathbf{r}_1 - \mathbf{r}_2|$ goes to infinity.

At far enough distances we have $U(\mathbf{r}_1 - \mathbf{r}_2)$ is constant, hence (by setting U=0 at infinity) $E = T_1 + T_2$ is constant:

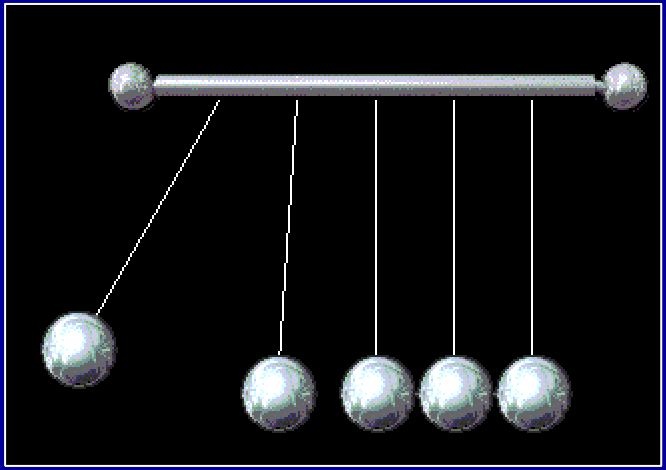
$T^{\text{well before}} = T^{\text{well after}}$, with $T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$.

Combined with the conservation of momentum $m_1 v_1 + m_2 v_2$, this allows us to easily analyze 2 particle collisions

Fact of pool table life [assuming equals mass and no head on collision]: After ball (1) has hit stationary ball (2), the angle between the two velocities afterwards is 90º.

Swinging Balls

The standard setup:
completely elastic collisions,
balls have equal mass,
negligible friction.



We know what happens when we let a ball swing with velocity v into the remaining four: its momentum and kinetic energy gets transferred to the rightmost ball, which will swing outward with the same velocity v.

Food for thought: why does it not happen that the two rightmost balls move outwards with velocity $\frac{2}{3}$ v, while the original ball returns with velocity $-\frac{1}{3}$ v ?

Energy of Several Particles

The generalization of the two particle result to several particles is straightforward as we only have to consider pair wise potentials: $U = U^{int} + U^{ext} = \sum_{\alpha,\beta>\alpha} U_{\alpha\beta} + \sum_{\alpha} U_{\alpha}^{ext}$
with $\mathbf{F}_{\alpha\beta} = -\nabla_{\alpha} U_{\alpha\beta}$.

For a conservative system, the total energy is thus:

$$E = \sum_{\alpha} \tfrac{1}{2} m_{\alpha} v_{\alpha}^{2} + \sum_{\alpha,\beta>\alpha} U_{\alpha\beta} + \sum_{\alpha} U_{\alpha}^{ext}$$

For rigid bodies, we have by definition that $|\mathbf{r}_{\alpha}-\mathbf{r}_{\beta}|$ is constant, hence *for central forces* $\mathbf{F}^{int}$, the potentials $U(|\mathbf{r}_{\alpha}-\mathbf{r}_{\beta}|)$ remain constant and can be ignored.

End of Midterm Material