# Classical Mechanics

## Phys105A, Winter 2007

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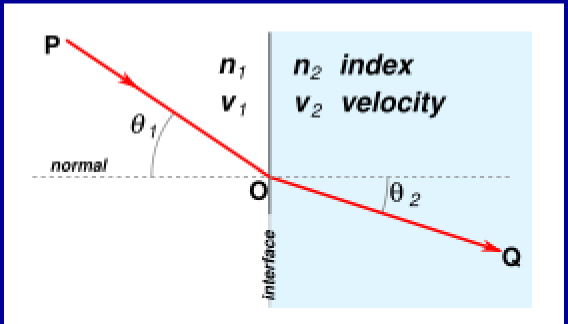
# Chapter 6: Calculus of Variations

**Many calculations in physics can be rephrased as minimization problems.**

**Fermat's principle**: When going from point P to Q, light takes that path that takes the least time.




"Given two media with different refractive indices, what is that path?"

# More Minimization

The natural, stable configuration of a necklace is that one that minimizes the potential energy of its chain.

"What is that shape?"

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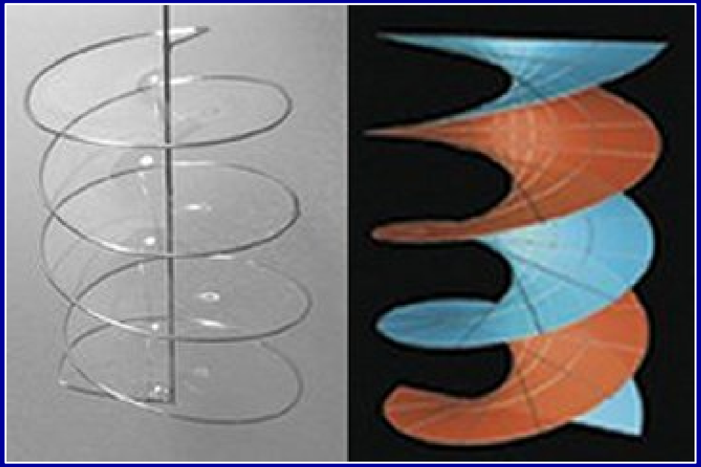
The stable shape of soap films is the one that minimizes its surface.

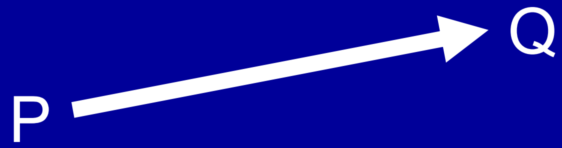
"For given boundary shapes, what is that soap shape?"



# Shortest/Fastest Path

Given two points P,Q in $\mathbb{R}^2$, what is the shortest path y(x)?

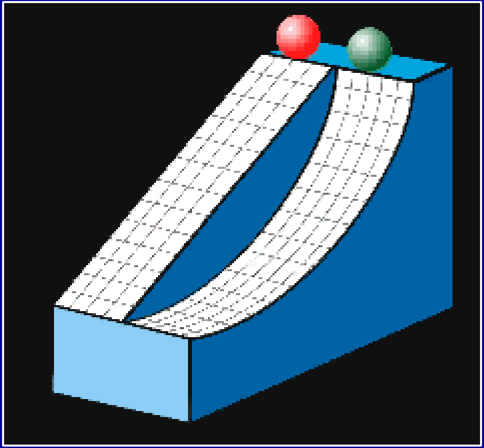
The path y(x) should minimize the integral: $\displaystyle\int_{x_1}^{x_2} \sqrt{1+y'^2}\,dx$




Given two points P,Q in $\mathbb{R}^2$ in a gravitational field g$\mathbf{e}_x$, what is the fastest path y(x)?

With the velocity v=√(2gx), the path y(x) should minimize the integral:

$$\frac{1}{\sqrt{2g}}\int_{x_1}^{x_2} \frac{\sqrt{1+y'^2}}{\sqrt{x}}\,dx$$



# Euler-Lagrange Equation

Let y(x) be the path that minimizes/maximizes the integral

$$S = \int_{x_1}^{x_2} f[y(x), y'(x), x]dx$$

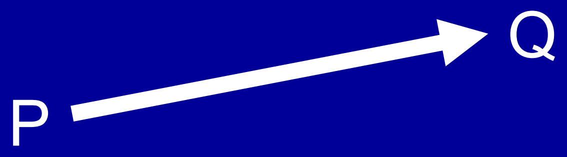
The Euler-Lagrange equation tells us that S is extremal when y(x) obeys

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$$

# Shortest Path, Answer

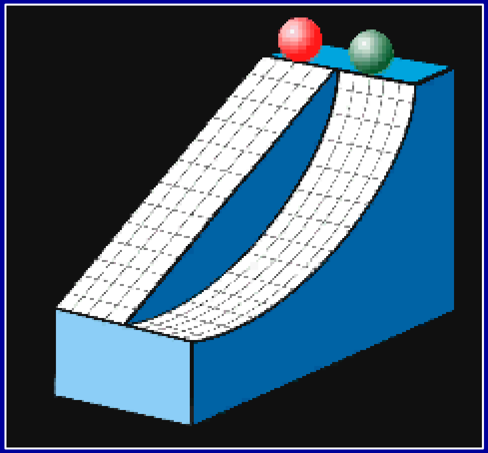
Given two points P,Q in $\mathbb{R}^2$, what is the shortest path y(x)?

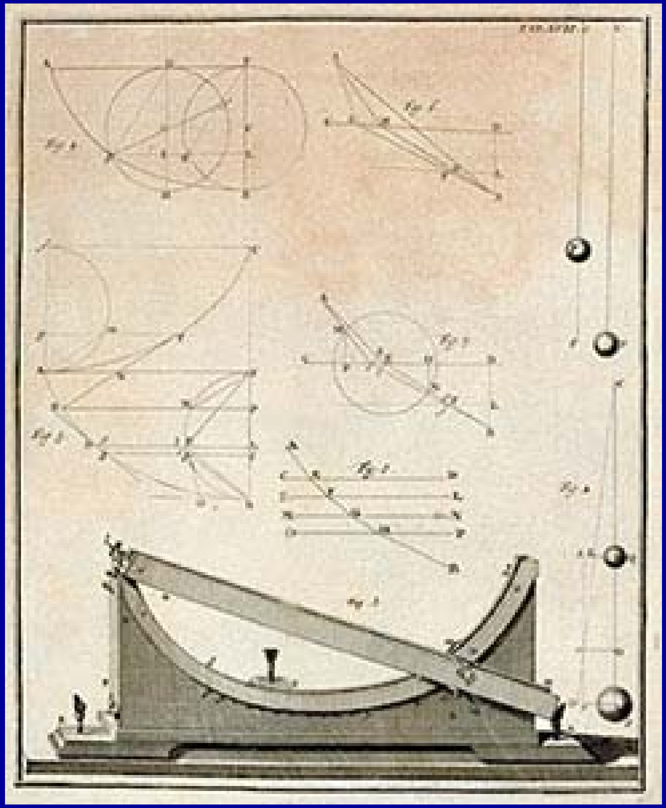
The path y(x) is a straight line (y' = dy/dx = constant).



# Shortest/Fastest Path II

Given two points P,Q in $\mathbb{R}^2$ in a gravitational field $g\mathbf{e}_x$, what is the fastest path y(x)?





The answer to this classic *brachistochrone problem* is that y(x) is (part) of a cycloid: