Classical Mechanics

Phys105A, Winter 2007

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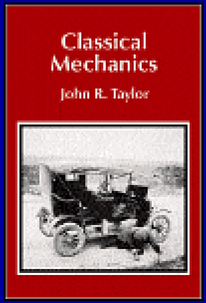
Formalities

- Latest news and course slides always found on the Phys105A site at http://www.cs.ucsb.edu/~vandam/…
- The required text book is John Taylor's "Classical Mechanics"



- TAs: Sungwoo Hong and Mariarosaria D'Alfonso
- Detailed homework and grading schedule TBA

Tentative Schedule

Monday:	11:30 Homework due (Broida, first floor)
Tuesday:	9:30–10:45 Class (Broida 1640)
	1:30–2:30pm: office hours WvD (room 5109)
Thursday:	9:30–10:45 Class (Broida 1640)
Friday:	1–1:50pm Discussion session (HSSB 1173)
Weekend:	Homework announced (due one week later)

Note that my office is in Harold Frank Hall (the old Engr. I)

HW, Midterm and Exam

Homework: Done on an individual basis. You have to able to convince others (like you) of your results. If your proof is sloppy or unnecessarily long, you will lose points.

Midterm+Exam: Much like homework but now with a time limit. Do no expect to get a 100% score.

The final grades are determined by "curving".

Physics 105A does not have projects.

Rules of the Game

Homework: Collaboration is fine, but it should be you alone who writes up the answers. You should be able to explain your answers when questioned.

If you are caught cheating you will get an "F". UCSB's misconduct policies will be strictly enforced.

Late homework policy: Homework that is late will not be accepted and thus gets a score 0.

Grading homework or exams for a second time can lead to a decrease in your grade.

Staying Up-To-Date

Frequently check the web site of the course at:
http://www.cs.ucsb.edu/~vandam/teaching/W07_Phys105A/

In case of exceptional situations:
email me as soon as possible.

When emailing me: make it clear that it is about Phys105A and sign your message.

Tentative Schedule

- At a rate of one chapter per week we should be able to cover Chapters 1–10 this quarter.
- The emphasis is on Newton's laws of motion, (angular) momentum, kinetic energy, oscillations, Euler-Lagrange equations and two body problems.

- Compared to your earlier encounters with classical mechanics the mathematics that we will use will be much more sophisticated.
- Linear algebra and differential calculus will be used extensively.

Central Theme

The “fundamental physical laws” that we use are not all that difficult and probably already familiar to you.

The meat of the course lies in the mathematical methods that we use to derive from these fundamental laws other physical laws that will allow us to perform calculations that are too difficult to do from ‘first principles’.

You should resist the temptation to think/believe that you can derive such laws by yourself “on the fly”.

Case in point: calculating the tides of the sea.

Chapter 1: Newton's Laws of Motion

Dealing with Space and Time

- Crucial to mechanics are the notions of space and time.
- To handle this in a mathematical way we need the mathematics of 3-dim, real valued vector spaces $\mathbb{R}^3$ and the ability to deal with differential equations.

- First some definitions and notation…

Vectors

- A 3 dimensional position vector can be denoted as:

$$\begin{aligned}\mathbf{r} &= \vec{r} \\ &= r_x\hat{x} + r_y\hat{y} + r_z\hat{z} \\ &= r_1 e_1 + r_2 e_2 + r_3 e_3 \\ &= (r_x, r_y, r_z)\ldots\end{aligned}$$

You are expected to be flexible enough to deal with different notations.

- Standard vector operations and constants
  - addition: $\mathbf{r}+\mathbf{s}=(r_x+s_x,r_y+s_y,r_z+s_z)$
  - multiplication with a scalar c: $c\mathbf{r}=(cr_x,cr_y,cr_z)$
  - zero vector: $\mathbf{0} = (0,0,0)$

Scalar/Dot Product

- Given two vectors **r** and **s,** we define the **scalar** or **dot** or **inner product** as **r•s** = $r_x s_x + r_y s_y + r_z s_z$.

- The length $r = \sqrt{(r_x^2+r_y^2+r_z^2)}$ of a vector **r** thus equals $r = |\mathbf{r}| = \sqrt{(\mathbf{r}\bullet\mathbf{r})} = \sqrt{(\mathbf{r}^2)}$.

- We have for the scalar product: **r•s** = rs cos θ with θ the angle between **r** and **s**.
- The nonzero vectors are orthogonal (**r**⊥**s**) if and only if **r•s**=0.

Vector/Cross Product

- For two vectors **r** and **s**, the **vector** or **cross product** **r**×**s** is a vector by itself and is defined by

$$\mathbf{r} \times \mathbf{s} = (r_y s_z - r_z s_y, r_z s_x - r_x s_z, r_x s_y - r_y s_x)$$

$$= \det \begin{bmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{bmatrix}$$

- Note that **r**×**s** = – **s**×**r**.
- The vector product only makes sense for 3d vectors.
- It holds that |**r**×**s**| = rs sin θ

Differential Calculus

- When describing a trajectory in space and time, we can use a time dependent position vector **r**(t).

- The **speed** of the object is then:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \left(\frac{dr_x}{dt}, \frac{dr_y}{dt}, \frac{dr_z}{dt}\right)$$

- The **acceleration** of the object is:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \left(\frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\right)$$

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = \ddot{\mathbf{r}} = \left(\frac{d^2r_x}{dt^2}, \frac{d^2r_y}{dt^2}, \frac{d^2r_z}{dt^2}\right)$$

Some Differential Rules

- Addition: d(**r**+**s**)/dt = d**r**/dt + d**s**/dt.
- Multiplication with scalar function f:
  d(f **r**)/dt = df/dt **r** + f d**r**/dt.
- What is d(**r•s**)/dt ?
- Answer: **r**•d**s**/dt + d**r**/dt•**s**

Two Exercises

- Exercise: Prove that **r**×**s** is orthogonal to **r** and **s**.
- Problem 1.10: Given the circular movement
  $\mathbf{r}(t) = R \cos \omega t\, e_x + R \sin \omega t\, e_y$
  What is the speed **v** and acceleration **a** of this particle?